3. Bond, Anchorage and Shear

- This chapter will discuss the following topics:
  
  - Outline the theory of calculating the anchorage bond length.
  - Determination of anchorage bond length, tension lap length and compression lap length by using Table 8.4 and Table 8.5 of the code.
  - Outline the design formulae for the design of shear reinforcement and the use of Table 6.3 of the Code.

3. Bond, Anchorage and Shear

- **Minimum Distance Between Bars**

  - The min. distance between bars are given in terms of $h_{agg}$ (the maximum size of aggregate).

  - Clear Horizontal and Vertical Distance between bars

    $\geq$ bar diameter or
    $\geq$ $(h_{agg} + 5\text{mm})$ or
    $\geq$ 20 mm whichever is greater.
3.1 Minimum and Maximum Percentages of Reinforcement in Beams, Slabs and Columns

- Minimum %: Table 9.1 of the code
- Maximum %
  (a) Beam (9.2.1.3)
    Neither the area of tension reinforcement nor the area of compression reinforcement should exceed 4% of the gross cross-sectional area of concrete.
  
  (b) Column (9.5.1)
    The longitudinal reinforcement should not exceed the following amounts, calculated as percentages of the gross cross-sectional area of the concrete:
    - vertically cast columns: 6%
    - horizontally cast columns: 8%
    - laps in the vertically or horizontally cast column: 10%

  (c) Wall (9.6.2)
    The area of vertical reinforcement should not exceed 4% of the concrete cross-sectional area of the wall.

3.2 Anchorage Bond

- The steel bar subjected to tension shown in Fig. 3.1 shall be firmly anchored in concrete. The anchorage length (embedded length) depends on the bond between the bar and the concrete, and the contact area.

![Fig. 3.1 Anchorage Bond](image)
3.2 Anchorage Bond

L = Min. anchorage length to prevent pull out.
φ = Bar size or nominal diameter.
f_{bu} = Ultimate anchorage bond stress.
f_{s} = Direct tensile or compressive stress in the bar.

Consider the forces on the bar,

Tensile pull-out force = Area of bar * Stress of bar
= \frac{\pi \phi^2}{4} \cdot f_s

Anchorage force = Contact Area * Bond Stress
= (L\pi\phi) \cdot f_{bu}

\therefore \frac{\pi \phi^2}{4} \cdot f_s = (L\pi\phi) \cdot f_{bu}

L = \frac{f_s}{4f_{bu}} \cdot \phi = \frac{0.87f_y}{4f_{bu}} \cdot \phi

The ultimate anchorage bond stress, \( f_{bu} = \beta \sqrt{f_{cu}} \).
Values of \( \beta \) are given in table 8.3.
\therefore anchorage length L can be written as
L = K_A \phi \quad \text{(see table 8.4 of the Code)}
3.2 Anchorage Bond

Table 8.3 - Values of the bond coefficient $\beta$

<table>
<thead>
<tr>
<th>Bar type</th>
<th>$\beta$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bars in tension</td>
<td>Bars in compression</td>
</tr>
<tr>
<td>Plain bars</td>
<td>0.28</td>
<td>0.35</td>
</tr>
<tr>
<td>Type 2 : deformed bars</td>
<td>0.50</td>
<td>0.63</td>
</tr>
<tr>
<td>Fabric</td>
<td>0.65</td>
<td>0.81</td>
</tr>
</tbody>
</table>

- Normally deformed type 2 bars are used in construction works.
- Furthermore anchorage may be provided by hooks and bends in the reinforcement. (see Fig. 8.2).

Fig. 8.2 Anchorage of Links

Figure 8.2 - Anchorage of links

Reproduce from HK Code
3.3 Lapping of Reinforcement

- Lapping of reinf. may be required if the reinf. is of insufficient length or there is curtailment (cutting off of reinf. when it is no longer required in order to save money) of reinf.

- Rules for lapping are:-
  - The laps should be staggered and be away from sections with high stresses. (i.e. avoid lapping of reinf. at sections with large bending moment as it induces large tensile stress in reinforcement)
  - Min. lap length should be not less than the greater of 15Φ or 300mm.
3.3 Lapping of Reinforcement

- Rules for lapping are:
  - Compression laps should be at least 25% greater than the compression anchorage length.
  - Lap lengths for unequal size bars may be based on the smaller bar.

If Cover < 2φ, multiple lap length by 1.4 or 2.0

![Diagram of lapping of tension reinforcement]

Fig. 3.3  Lapping of tension reinforcement

3.4 Shear

- Shear reinforcement can be in the form of shear links and inclined bars (bent-up bars). However, inclined bars are less frequently used in construction today.

- **Shear Links (Stirrups)**
  - Links can be broadly classified into shear link and nominal link (minimum link) and the design formulae for these types of links are as follows:
Design Formulae for Shear Reinforcement for $f_{cu} \leq 40$ MPa

- Shear Links:-
  \[
  \frac{A_{sv}}{s_v} \geq \frac{b(v - v_c)}{0.87 f_{yy}}
  \]

- Minimum (nominal) Links:-
  \[
  \frac{A_{sv}}{s_v} \geq \frac{0.4b}{0.87 f_{yy}}
  \]

Where:-

- $A_{sv}$ = Total cross-sectional area of the vertical legs of the link.
- $b$ = Breadth of beam
- $f_{yy}$ = Characteristic strength of the shear reinf. = $f_y$
- $s_v$ = Longitudinal spacing of shear links
- $v$ = Design shear stress
  - = Design shear force/(bd) = $V/(bd)$
- $v_c$ = The ultimate shear stress that can be resisted by the concrete.
Design Formulae for Shear Reinforcement

The values of $v_c$ is given in table 6.3 of the Code.

- It shall be noted that the values given in table 6.3 is for grade 25 conc. For other grades of conc., adjustment shall be made by multiplying $\left( \frac{f_{cu}}{25} \right)^{\frac{1}{3}}$

- The values of $v_c$ increases for shallow members and those with larger percentages of tensile reinforcement.

3.5 Enhanced Shear Resistance

- Within a distance of $2d$ from a support or a concentrated load, the design concrete shear stress $v_c$ may be increased to

  $$v_c \cdot \left( \frac{2d}{a_v} \right)$$

- The distance $a_v$ is measured from the support or concentrated load to the section being designed.

  – Average shear stress should never exceed the lesser of $0.8\sqrt{f_{cu}}$ or 7 N/mm².
Figure 6.3 – Shear failure near supports

Note: The shear causing failure is that acting on section x-x.

Table 6.3 - Values of $v_c$ Design Concrete Shear Stress

<table>
<thead>
<tr>
<th>$100d_l$</th>
<th>$h/b_d$</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
<th>225</th>
<th>250</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>N/mm²</td>
<td>N/mm²</td>
<td>N/mm²</td>
<td>N/mm²</td>
<td>N/mm²</td>
<td>N/mm²</td>
<td>N/mm²</td>
<td>N/mm²</td>
</tr>
<tr>
<td>&lt;0.15</td>
<td>0.40</td>
<td>0.40</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>0.25</td>
<td>0.50</td>
<td>0.50</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
<td>0.48</td>
</tr>
<tr>
<td>0.50</td>
<td>0.66</td>
<td>0.66</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>0.75</td>
<td>0.77</td>
<td>0.77</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
<td>0.71</td>
</tr>
<tr>
<td>1.00</td>
<td>0.85</td>
<td>0.85</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td>1.50</td>
<td>0.97</td>
<td>0.97</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>2.00</td>
<td>1.06</td>
<td>1.06</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>≥3.00</td>
<td>1.22</td>
<td>1.16</td>
<td>1.12</td>
<td>1.06</td>
<td>1.05</td>
<td>1.02</td>
<td>0.98</td>
<td>0.91</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Allowance has been made in these figures for a $f_{cd}$ of 1.25.
2. The values in the table are derived from the expression:
   
   $0.9\left(\frac{100d_l}{h/b_d}\right)^{0.5}\left(\frac{400}{d}\right)^{0.1}$

   where:
   
   $\frac{100d_l}{h/b_d}$ should not be taken as greater than 3.
   
   $\left(\frac{400}{d}\right)^{0.5}$ should not be taken as less than 1 and for members without shear reinforcement.
   
   $\left(\frac{400}{d}\right)^{0.1}$ should not be taken as less than 1 for members with shear reinforcement providing minimum links in accordance with Note 1 in Table 6.2.

3. For characteristic concrete strengths greater than 25 N/mm², the values in the table may be multiplied by $\left(\frac{f_{cd}}{25}\right)^{0.5}$. The value of $f_{cd}$ should not be taken as greater than 80 N/mm².

Reproduce from HK Code
### Table 9.1 - Minimum Percentage of Reinforcement

<table>
<thead>
<tr>
<th>Situation</th>
<th>Definition of percentage</th>
<th>Minimum percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$f_y = 250$ N/mm² (%)</td>
</tr>
<tr>
<td>Tension reinforcement</td>
<td>$100L_d/L_c$</td>
<td>0.6</td>
</tr>
<tr>
<td>Sections subjected mainly to pure tension</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.32</td>
</tr>
<tr>
<td>Sections subjected to flexure:</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.24</td>
</tr>
<tr>
<td>(i) flanged beams, web in tension:</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.48</td>
</tr>
<tr>
<td>(ii) flanged beams, flange in tension:</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.36</td>
</tr>
<tr>
<td>(iii) rectangular section</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.24</td>
</tr>
<tr>
<td>Compression reinforcement</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.4</td>
</tr>
<tr>
<td>(where such reinforcement is required for the ultimate limit state)</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.2</td>
</tr>
<tr>
<td>General rule</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.4</td>
</tr>
<tr>
<td>Simplified rules for particular cases:</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.2</td>
</tr>
<tr>
<td>(i) rectangular beam</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.2</td>
</tr>
<tr>
<td>(ii) flanged beam</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Flange in compression</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Web in compression</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Transverse reinforcement in flanges of flanged beams (provided over full effective flange width near top surface to resist horizontal shear)</td>
<td>$100L_{w,h} / L_{w,h}$</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Notes:
- The minimum percentages of reinforcement should be increased where necessary to meet the ductility requirements given in clause 9.9.

Reproduce from HK Code

### Table 8.4 – Ultimate Anchorage Bond Lengths ($l_b$)

<table>
<thead>
<tr>
<th>Concrete Grade</th>
<th>Type of anchorage length</th>
<th>$f_y$ 250 N/mm²</th>
<th>$f_y$ 460 N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Tension Compression</td>
<td>36</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>35</td>
<td>Tension Compression</td>
<td>33</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>40</td>
<td>Tension Compression</td>
<td>31</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>45</td>
<td>Tension Compression</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>50</td>
<td>Tension Compression</td>
<td>28</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>22</td>
<td>23</td>
</tr>
<tr>
<td>$\geq 60$</td>
<td>Tension Compression</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 8.4 - Ultimate anchorage bond lengths ($l_b$) as multiples of bar diameter

Reproduce from HK Code
### Table 8.5 Ultimate Lap Lengths

<table>
<thead>
<tr>
<th>Concrete Grade</th>
<th>Type of lap length</th>
<th>$f_y$ 250 N/mm²</th>
<th>$f_y$ 460 N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tension and compression lap length – $l_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.4 x tension lap</td>
<td>36</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>2.0 x tension lap</td>
<td>50</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>71</td>
<td>74</td>
</tr>
<tr>
<td>35</td>
<td>1.4 x tension lap</td>
<td>33</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>2.0 x tension lap</td>
<td>46</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>66</td>
<td>68</td>
</tr>
<tr>
<td>40</td>
<td>1.4 x tension lap</td>
<td>31</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>2.0 x tension lap</td>
<td>43</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>62</td>
<td>64</td>
</tr>
<tr>
<td>45</td>
<td>1.4 x tension lap</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>2.0 x tension lap</td>
<td>41</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>58</td>
<td>60</td>
</tr>
<tr>
<td>50</td>
<td>1.4 x tension lap</td>
<td>28</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>2.0 x tension lap</td>
<td>39</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>55</td>
<td>57</td>
</tr>
<tr>
<td>≥ 60</td>
<td>1.4 x tension lap</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>2.0 x tension lap</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>61</td>
<td>52</td>
</tr>
</tbody>
</table>

Notes:
1. The values are rounded up to the nearest whole number and the length derived from these values may differ slightly from those calculated directly for each bar or wire size.

*Table 8.5 - Ultimate lap lengths as multiples of bar diameter*

---

### Figure 8.5 - Factors for lapping bars

**Top bars**

- Factor: 2
- Factor: 2
- Factor: 1.4
- Factor: 1.0
- Factor: 1.4

**Bottom bars**

- Factor: 1.4
- Factor: 1.4
- Factor: 1.0
- Factor: 1.4

*Note: For laps in bottom of section as cast minimum cover criteria applies to corner bars only

\[ s = 75 \text{mm or } 6\theta, \] whichever is greater

\[ \theta = \text{diameter of the lapped reinforcement} \]
Self-Assessment Questions

Q1. Determine the anchorage bond length from the first principle for a T32 bar in tension (deformed type 2) and $f_{cu} = 35 \text{ N/mm}^2$. The bars as listed below:

Choices:
(a) 1000 mm
(b) 1082 mm
(c) 1360 mm

Self-Assessment Questions

Q2. Determine the anchorage bond length by using table 8.4 of HKC 2004 for a R16 bar in compression and $f_{cu} = 30 \text{ N/mm}^2$.

Choices:
(a) 464 mm
(b) 528 mm
(c) 540 mm
Self-Assessment Questions

Q3. Determine the ultimate shear resistance of concrete $v_c$ for the singly reinforced concrete section shown in Fig. Q3, without shear enhancement. Given that $f_{cu} = 35$ N/mm$^2$ and $f_y = 460$ N/mm$^2$.

Choices:

(a) 0.70 N/mm$^2$

(b) 0.90 N/mm$^2$

(c) 1.10 N/mm$^2$

Assignment No. 3

AQ1  Fig. AQ1 shows the lapping of steel reinforcement of a beam. Determine the lap length for the bar groups 1, 2, 3 and 4. Given that the nominal cover is less than 2 times the diameter of the bar and $f_{cu} = 35$ N/mm$^2$. The spacing between group 1 and 2 bars is greater than $s$ and the spacing between group 3 & 4 bars is less than $s$.

(a) The lapping of steel reinforcement is at tension zone.

(b) The lapping of steel reinforcement is at compression zone.
Assignment No. 3

AQ2 Determine the nominal cover and breadth of the web for the simply supported beams whose sections are shown in Fig. AQ2a, AQ2b & AQ2c. The maximum size of the aggregate in each case is 20 mm, $f_{cu} = 40 \text{ N/mm}^2$ and the link size is 10mm (min. cement content $= 300 \text{ kg/m}^3$ and max water-cement ratio = 0.55).

(a) Exposure condition: severe
    Fire resistance: 2 hours

(b) Exposure condition: moderate
    Fire resistance: 1 hours

(c) Exposure condition: severe
    Fire resistance: 4 hours
Assignment No. 3

AQ3. Determine the average shear stress for the section shown in Fig. AQ3a, AQ3b & AQ3c, and state whether shear reinforcement is required. Given that $f_{cu} = 30$ N/mm$^2$ and $f_y = 460$ N/mm$^2$.

![Fig. AQ3a](image1)
![Fig. AQ3b](image2)
![Fig. AQ3c](image3)

Veterinary Shear = 150 kN
Veterinary Shear = 60 kN
Veterinary Shear = 250 kN

Assignment No. 3

AQ4. The sections in Fig. AQ4a & AQ4b are subject to shear forces at the ultimate limit state as shown. Determine the spacing of R10 links. Given that $f_{cu} = 35$ N/mm$^2$, $f_y = 460$ N/mm$^2$ and $f_{yv} = 250$ N/mm$^2$.

![Fig. AQ4a](image4)
![Fig. AQ4b](image5)

Veterinary Shear = 40 kN
Veterinary Shear = 160 kN