CHAPTER 6

Design of R C Columns and Walls

Learning Objectives

- Classify columns for design
- Design R C short braced columns for uniaxial and biaxial loads
- Extend the design method to design R C walls

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6.1 Columns

A column is generally referred to a vertical member designed to resist mainly compression. In building construction, columns are usually constructed monolithically with beams to form an integral structural frame. Hence, a column will inevitably, in addition to compression, have to take up bending moment transmitted from beams to which it is connected. Therefore, unlike beam section, which is designed solely for bending, column section has to be designed for combined action of axial compression, \( N \), and moment, \( M \).

In this chapter, the basic assumptions for beam in Chapter 2 will be adopted to derive the design charts for column, and then you will learn how to make use of these design charts to design column sections and then extend this method to design RC walls.

Before embarking on the calculation to determine the amount of steel required for a column section, the design forces have to be established. In addition to the forces applied directly to the member or obtained by elastic analysis, when a member is under compression, additional bending may be induced when it deflects laterally as shown in the figure on the right.

There are two types of lateral deflection. The first one is the result of side sway of the whole structural frame, \( \Delta \), and the second one is due to bending of the column itself, \( \delta \). They will then induce additional moments, \( N\Delta \) and \( N\delta \) respectively to the column.

If the column is too slender, this additional moment will induce further lateral deflection, and then further additional bending and so on leading to...
sideway buckling failure of the column that is sudden and catastrophic.

If this lateral deflection is restrained and if the column is sturdy and stiff enough to resist further propagation of lateral deflection and bending, this additional moment may be small and can even be neglected in the design. This is the case when the structure is braced against side sway and the column is not slender, which is called short braced column. The design of this type of column is the focus of this Chapter.

### 6.1.1 Classification of Columns

According to the structural behaviour of the column in a structure as described above, columns can be classified into:

(a) Braced or non-braced

If a structure is provided with wall or bracing to resist lateral forces, the columns of this structure can then be regarded as braced, and the lateral deflection due to side sway of the structure would then be so small that additional moment induced by side sway, or $\Delta$-effect, can be ignored in the design.\(^1\)

**Example**

For the structural framing plan as illustrated in the following figure, lateral loads in the Y-direction will be resisted by shear walls at both ends of the structure and therefore the columns are considered as braced in the Y-direction. However, as the bending stiffness of the wall in the X-direction is comparatively small, the columns will deflect under lateral loads in the X-direction and therefore the columns are considered as non-braced in the X-direction.

---

\(^1\) Otherwise, reference shall be made to Cl. 6.2.1.3 of HKCP-2013 for additional design moments. It is beyond the scope of the chapter.
(b) Short or slender

For braced column, if the effective height to width ratio of the column is less than 15, the column can then be regarded as short, and additional moment due to lateral deflection of the column, or $\delta$-effect, can be ignored in the design.\(^2\)

Short braced column:  $L_{ex} / h$ and $L_{ey} / b < 15$

Take note that effective height, $L_e$, instead of overall height is used in defining the slenderness of a column. It is given by:

$$L_e = \beta L_o \quad [6.1]$$

where

$L_o$ = The clear height between end restraints

The value of $\beta$ is given in the following table:

\(^2\) Otherwise, reference shall be made to Cl. 6.2.1.3 of HKCP-2013 for additional design moment. For non-braced column, this ratio is reduced to 10.
The end conditions are defined as follows (extracted from Cl. 6.2.1.1(e)ii of HKCP-2013):³

Condition 1. The end of the column is connected monolithically to beams on either side which are at least as deep as the overall dimension of the column in the plane considered. Where the column is designed to a foundation structure, this should be of a form specifically designed to carry the moment.

Condition 2. The end of the column is connected monolithically to beams on either side which are shallower than the overall dimension of the column in the plane considered.

Condition 3. The end of the column is connected to members which, while not specifically designed to provide restraint to rotation of the column will, nevertheless, provide some nominal restraint.

### 6.1.2 Example – Effective Height to Width Ratio

**Question**

A braced column is shown in the figure below. Beams are connected monolithically to it. With the information given, determine whether the column is short or slender.

³ Value of $\beta$ for non-braced column is in general larger than that for braced column. Refer to Table 6.12 of HKCP-2013 for details.
Column: \( h = 550 \text{ mm (in Y-direction)} \)  
\[ b = 450 \text{ mm} \]  
Floor-to-floor height = 7000 mm

Beams at the Top
Depth of the beam in X-direction = 400 mm  
Depth of the beam in Y-direction = 425 mm

Beams at the Bottom
Depth of the beam in X-direction = 500 mm  
Depth of the beam in Y-direction = 475 mm

Solution

Plane in the X-Direction (bending about Y-axis)

<table>
<thead>
<tr>
<th>Beam depth/column width</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top end 400/450 &lt; 1</td>
<td>2</td>
</tr>
<tr>
<td>Bottom end 500/450 &gt; 1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \beta = 0.80 \]
\[ L_{oy} = 7000 - 400 = 6600 \text{ mm} \]
\[ L_{oy} = 0.80 \times 6600 = 5280 \text{ mm} \]

Effective height to width ratio, \( L_{oy} / b = 5280 / 450 \)
\[ = 11.7 < 15 \text{ (Short column)} \]
Plane in the Y-Direction (Bending about X-axis)

<table>
<thead>
<tr>
<th>Beam depth/column width</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top end 425/550 &lt; 1</td>
<td>2</td>
</tr>
<tr>
<td>Bottom end 475/550 &lt; 1</td>
<td>2</td>
</tr>
</tbody>
</table>

$\beta = 0.85$

$L_{ox} = 7000 - 425 = 6575 \text{ mm}$

$L_{ox} = 0.85 \times 6575 = 5589 \text{ mm}$

Effective height to width ratio, $L_{ox}/h = 5589 / 550$

$= 10.2 < 15$ (Short column)

Therefore, it is a short braced column.

### 6.1.3 Design Forces

In general, structural analysis is required to obtain the design forces to design a column section. The design code requires that the structure has to be analyzed by appropriate load arrangements to obtain the following critical combinations of design axial forces and moments (Cl.5.1.3.3 of HKCP-2013):

i. maximum axial load combined with co-existent bending moment,

ii. minimum axial load combined with co-existent bending moment,

iii. maximum bending moment combined with co-existent axial load, and

iv. any another co-existent combination of axial load and bending moment which will be more critical to the column design than the above cases.

In the absence of rigorous structural analysis, the design code allows the following simplified methods to obtain the design forces:

(a) The design axial force may be calculated by assuming slabs and beams simply supported (Cl.6.2.1.2(c) of HKCP2013); or, in other words, by tributary area method as illustrated in the following figure.
Figure 6.1 – Tributary Area for Determination of Column Load (Plan)

(b) If a column, by nature of the structure, is not subjected to significant moments, the design moment can be obtained by using *nominal eccentricity*, $e_{\text{min}}$ (Cl.6.2.1.2 of HKCP-2013):

$$e_{\text{min}} = \text{minimum (0.05 x depth of the column, 20mm)} \quad [6.2]$$

$$M = N e_{\text{min}}$$

This is the *nominal design moment* for column; or, in other words, the design moment of a column should not be taken as less than this value.

(c) Design moment may be obtained by *simplified sub-frame analysis* in accordance with Cl.5.2.5.3 of HKCP-2013 as illustrated in the following figure.
\[ K = \text{Stiffness of the member} \]
\[ K = 4E(bh^3/12)/L \text{ (for rectangular beam with fixed end)} \]
\[ M_{\text{unbal}} = \text{The unbalance fixed end moment (FEM) of beams} \]
\[ = (\text{FEM of } b_1) - (\text{FEM of } b_2) \]
\[ \text{FEM} = (1/12)wL^2 \text{ (for beam under udl)} \]
\[ M_{\text{cl}} = \text{Moment to the lower column} \]
\[ = M_{\text{unbal}} K_{\text{cl}} / (K_{\text{cl}} + K_{\text{cu}} + 0.5K_{b1} + 0.5K_{b2}) \]
\[ M_{\text{cu}} = \text{Moment to the upper column} \]
\[ = M_{\text{unbal}} K_{\text{cu}} / (K_{\text{cu}} + K_{\text{cl}} + 0.5K_{b1} + 0.5K_{b2}) \]

**Figure 6.2 – Simplified Sub-frame Analysis for Determination of Design Moment for Column**

**Notes to Figure 6.2**

i. The remote ends of the members are assumed to be fixed; unless, it is clearly more reasonable to be pinned.

ii. Beam of longer span, \( b_1 \), is maximum loaded while that of the shorter span, \( b_2 \), is minimum loaded to obtain the maximum unbalanced moment.

iii. For rectangular beam with fixed end, \( K = 4E(bh^3/12)/L \), where \( b = \text{breadth of the section} \), \( h = \text{overall depth of the section} \), \( (bh^3/12) = \text{moment of inertia of the section} \), \( L = \text{length of the element} \). As the Youngs' Modulus of the material, \( E \), is constant, it can be omitted.

iv. In the formulae of moment distribution, the stiffness of beams is reduced by 0.5 to account for the decrease in stiffness when the beam cracks under bending.

### 6.1.4 Example – Reduction of Design Loads

If a column carries loads from more than one floor, the total distributed imposed load on it may be reduced by 5% per additional floor up to the maximum of 40%. Details of this allowance shall refer to Cl.3.7 of the Code of Practice for Dead and Imposed Loads – 2011.
**Question**

A 600 x 600 short braced column supports beams in symmetrical arrangement.

The tributary area for the column is 95 m².

Dead load, \( g_k = 5.3 \text{ kPa} \) (assuming self-weight of the column included).

Imposed load, \( q_k = 3.0 \text{ kPa} \).

Number of floors supported = 10.

Determine the design axial force for the column.

**Solution**

Allowable percentage reduction = \((10-1) \times 5\% = 45\% > 40\% \) (use 40%)

Dead Load, \( G_k = 5.3 \times 95 \times 10 \)

\[= 5,035 \text{ kN} \]

Imposed load, \( Q_k = 3.0 \times 95 \times 10 \times (1 - 40\%) \)

\[= 1,710 \text{ kN} \]

Design axial load, \( N = 1.4 \times 5035 + 1.6 \times 1710 \)

\[= 9,785 \text{ kN} \]

(Comment: If there is no reduction, the total design axial load is 11,609 kN. Although the reduction in imposed load is 40%, the reduction in total design axial load is only 16%.)

**6.1.5 Example – Design Moment by Simplified Sub-frame Analysis**

**Question**

A braced short column supports beams B1 and B2 on each side and an upper column with the following design parameters. Determine the maximum design moment transmitted from the beams.

- Column dimensions \( b = 350 \text{ mm} \)
- \( h = 400 \text{ mm} \) (in the direction of the beams)
- Floor-to-floor height, \( L_c = 3500 \text{ mm} \) (same for upper and lower columns)
- Beam dimensions \( h \times b = 700 \times 300 \text{ mm} \) (same for B1 & B2)
- Span of B1, \( L_{b1} = 9700 \text{ mm} \)
- Span of B2, \( L_{b2} = 5000 \text{ mm} \)
- Loading on the beams B1 & B2
  - Dead Load, \( g_k = 30.16 \text{ kN/m} \) (beam self-weight included)
  - Imposed Load, \( q_k = 53.25 \text{ kN/m} \)
Solution

Max design load for beam = 1.4 \times 30.16 + 1.6 \times 53.25
= 127.4 kN/m

Min design load for beam = 1.0 \times 30.16
= 30.2 kN/m

FEM_{B1} = \frac{1}{12} \times 127.4 \times 9.7^2
= 999 kN-m

FEM_{B2} = \frac{1}{12} \times 30.2 \times 5.0^2
= 63 kN-m

Unbalanced moment, M_{un} = 999 - 63
= 936 kN-m

Moment Distribution

K_{cl} = K_{cu} = \frac{4 \times (1/12 \times 350 \times 400^3)}{3500}
= 2.133 \times 10^6 \text{ mm}^3

0.5K_{b1} = \frac{2 \times (1/12 \times 300 \times 700^3)}{9700}
= 1.768 \times 10^6 \text{ mm}^3

0.5K_{b2} = \frac{2 \times (1/12 \times 300 \times 700^3)}{5000}
= 3.430 \times 10^6 \text{ mm}^3

Moment Distribution Factor = \frac{2.133}{(2 \times 2.133 + 1.768 + 3.430)}
= 0.225

Therefore,

Design moment to column, M = 936 \times 0.225
= \textbf{211 kN-m}
6.2 Design of Column Section

6.2.1 Design Formulae for Axially Loaded Column

(a) Short Column Subjecting to Axial Load with Moment due to Nominal Eccentricity

If a column, due to the nature of the structure, cannot be subjected to significant moments, it may be designed by Eqn 6.55 of HKCP-2013:

\[ N = 0.4f_{cu}A_c + 0.75f_yA_{sc} \]  \[\text{[6.3]}\]

This formula is useful for design of columns, which are, in theory, subjected to axial load only, such as load transferred to the column by centrally-positioned bearing, or columns supporting very stiff structure or very deep beam. Comparing with the ultimate capacity, \( N_{uz} = 0.45f_{cu}A_c + 0.87f_yA_{sc} \), it is reduced by about 10% to account for the nominal eccentricity of 0.05h.

(b) Short Braced Column Supporting an Approximately Symmetrical Arrangement of Beams

If the spans of the beams supported on both sides of a column are approximately equal, i.e. not differ by more than 15% of the longer, and the design loads are uniformly distributed, the column can be designed by Eqn 6.56 of HKCP-2013:

\[ N = 0.35f_{cu}A_c + 0.67f_yA_{sc} \]  \[\text{[6.4]}\]

The axial load capacity given by this formula deems to have taken into account the moment induced by asymmetric loading of the beams.

Note that the above two formulae have included an allowance for \( \gamma_m \).
6.2.2 Example – Design of Short Braced Column for Axial Load

**Question**

Design the main reinforcement for the column in the example of 6.1.4 with the following design parameters.

- Column dimensions: \( h = b = 600 \text{ mm} \)
- Grade of concrete, \( f_{cu} = 40 \text{ MPa} \)
- Reinforcement bars, \( f_y = 500 \text{ MPa} \)

**Solution**

Design axial compression, \( N = 9,785 \text{ kN} \) (from 6.1.4)

As the column is supporting symmetrical arrangement of beams

Eqn 6.56 of HKCP-2013 is adopted.

\[
N = 0.35 f_{cu} A_c + 0.67 f_y A_{sc}
\]

\[
0.67 f_y A_{sc} = 9,785 \times 10^3 - 0.35 \times 40 \times 600 \times 600
\]

\[
= 4745 \times 10^3 \text{ N}
\]

\[
A_{sc} = \frac{4745 \times 10^3}{(0.67 \times 500)}
\]

\[
= 14,164 \text{ mm}^2
\]

(Provide 8T40 + 8T32)

\[
A_{sc, pro} = 8 \times 1257 + 8 \times 804 = 16,488 \text{ mm}^2
\]

\[
100A_{sc, bh} = 4.6\%
\]

> 0.8\% and < 6\% (Steel ratio ok)

Limits to steel ratio for column refer to Table 2.1 in Chapter 2 or the Annex.

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COLUMN SECTION
6.2.3 Design Charts for Combined Axial and Uniaxial Moment

When a symmetrically-reinforced rectangular section is subjected to combined axial force and moment, the strain and stress distribution is shown in the following figure. The assumptions for the analysis of beam section in Chapter 2 are still applicable.

![Diagram of column section](image)

Figure 6.3 – Strain and Stress Distribution of Column Section

Given the values of b, h, d, \( f_y \) and \( f_{cu} \) of the section, assuming an amount of steel, \( A_s \), try with a value of the depth of neutral axis, \( x \), and then calculate the values of \( M \) and \( N \) by the following steps:

(i) By compatibility of strain, with the crushing strain of concrete \( \varepsilon_{cu} = 0.0035 \), calculate the strains of top and bottom bars, \( \varepsilon_{s1} \) & \( \varepsilon_{s2} \).

(ii) Calculate the stresses of the bars by using the stress-strain relation of steel and then the forces, \( F_{s1} \) and \( F_{s2} \) in the bars.

(iii) Calculate the compression force, \( F_{cc} \), in the concrete by using simplified stress block.

(iv) By equilibrium of axial forces and moments, the values of \( N \) and \( M \) can then be obtained.
Repeat the above steps for another value of $x$, and so on an $M$-$N$ interaction curve can then be plotted as shown below.

When the value of $x$ is very small, the strain of tension bars at the bottom of the section is very large as illustrated in the figure on the right. Bottom bars have yielded before the concrete crushes. Bending is dominating and the section will fail in flexural tension. The values of $M$ & $N$ fall on the lower part of $M$-$N$ curve between points "a" and "b".

When the value of $x$ increases, the strain of the bottom bars decreases and it will reach a point at which the bottom bars yields at the same time when concrete crushes. It is point "b" on the $M$-$N$ curve and is called balanced failure. When the value of $x$ increases beyond this point of balance, the section will fail by crushing of concrete before yielding of tension steel. Axial compression becomes dominating.
When the value of $x$ increases further, the compression in the top bars increases and then yields at point "c" of the M-N curve. On the other hand, the tension in bottom steel decreases and then equal to zero at point "d" on the M-N curve where the value of $x$ equals to $d$, that means the neutral axis is at the centroid of the bottom steel, and beyond which the bottom bars changes from tension to compression.

When the value of $x$ is larger than $h$, i.e. beyond point "e" on the M-N curve, the whole section is then in compression. The ultimate value of $x$ is reached when both top and bottom bars yield in compression and, at this point, the moment becomes zero and the section is at the ultimate axial capacity, $N_{uz}$.

The M-N interaction curve represents the capacity of the section reinforced with an amount of steel, $A_s$. If the point of coordinates of the design forces, $(N, M)$, falls within the curve, that means the section is stronger than enough to resist the design forces, and vice versa. Hence, it is desirable to identify a value of $A_s$ of which the N-N curve just touches and embraces the point of the design forces.

By altering the value of $A_s$, a set of M-N curves in the form of design chart can be derived to facilitate the determination of the amount of steel required for a column section. Instead of $d$, $N$, $M$ and $A_s$, the following dimensionless parameters are used to derive a set of generalized design charts.

$$
\begin{align*}
\frac{d}{h} & \quad \frac{N}{bf_{cu}} & \quad \frac{M}{bh^2f_{cu}} & \quad \frac{A_sf_y}{bf_{cu}}
\end{align*}
$$

A set of five typical Column Design Charts and the notes on their derivation are attached at the end of this chapter for reference.
The steps in using the Column Design Charts for design are as follows:

i. Estimate the value of $d/h$, round-down to the nearest 0.5, and then select the Chart with corresponding $d/h$ value.

ii. Calculate the values of $N/(bh f_{cu})$ and $M/(bh^2 f_{cu})$ and plot it on the Chart.

iii. Linear interpolate the value of $A_s f_y/(bh f_{cu})$ from the two adjacent curves.

For example,
If $d/h = 0.825$, use the Design Chart with $d/h = 0.80$.
If $N/(bh f_{cu}) = 0.45$ and $M/(bh^2 f_{cu}) = 0.094$, the value of $A_s f_y/(bh f_{cu})$ read from the Design Chart is 0.34, as illustrated below.4

4 A more rigorous approach is to find also the value of $A_s f_y/(bh f_{cu})$ from the Chart with $d/h = 0.85$, which is 0.30 in this case, and then calculate the value of $A_s f_y/(bh f_{cu})$ corresponding to $d/h = 0.825$ by linear interpolation, i.e. $(0.30 + 0.34)/2 = 0.32$. Nevertheless, the value of 0.34 is on the safe side and the difference is only 6%.
6.2.4 Examples – Design of Short Braced Column under Uniaxial Loads

Question A

DWG-06 shows a part plan and the elevation of a braced building structure. Determine the main reinforcement bars for column C2 at Ground Floor by considering the loading arrangement that will create the maximum bending moment to the column.

Design parameters

Column C2
- Column breadth, \( b = 350 \) mm (in X-dir)
- Column depth, \( h = 400 \) mm
- Floor-to-floor height = 3500 mm (same for all floors)

Beams in Y-direction
- Beams BY1 & BY2, \( h \times b = 700 \times 300 \) mm
  - Beam BY1 span, \( L_{by1} = 9900 - 400/2 = 9700 \) mm
  - Beam BY2 span, \( L_{by2} = 5000 \) mm

Beams in X-direction
- Beams BX1 & BX2, \( h \times b = 500 \times 300 \) mm
  - Beam BX1 span, \( L_{bx1} = 3600 \) mm
  - Beam BX2 span, \( L_{bx2} = 3500 \) mm

- Slab thickness = 225 mm (same for all floors)
- Allowance for finishes = 2.0 kPa (same for all floors)
- Imposed load = 15.0 kPa (same for all floors)
- No of storeys = 3
  - \( f_{cu} = 40 \) MPa
  - \( f_y = 500 \) MPa
- Cover = 40 mm
- Preferred link size = 10
- Preferred bar size = 40

Solution

The column is obviously a short column because even the floor-to-floor height to width ratio, i.e. \( 3500/350 = 10 << 15 \).

Loading to Beams in Y-Directions

Load width = \( (3600 + 3500)/2 = 3550 \) mm
Dead Load
Slab \(24.5 \times 0.225 \times 3.550 = 19.57\text{ kN/m}\)
Finishes \(2.0 \times 3.550 = 7.10\text{ kN/m}\)
Beams \(24.5 \times 0.3 \times 0.475 = 3.49\text{ kN/m}\)
\[\text{30.16 kN/m}\]
Imposed Load \(15.0 \times 3.550 = 53.25\text{ kN/m}\)

**Loading to Beams in X-Direction**
(No load transferred from slab)
Dead Load only
Beams self-weight \(24.5 \times 0.3 \times 0.275 = 2.02\text{ kN/m}\)

**Self-weight of Column**
Dead Load only \(24.5 \times 0.35 \times 0.40 \times 3.5 = 12.00\text{ kN per floor}\)

**Axial Load from the Floors Above 1/F**
Dead Load
Beams in Y-Dir \(2 \times 30.16 \times (9.7 + 5.0)/2 = 443\text{ kN}\)
Beams in X-Dir \(2 \times 2.02 \times (3.6 + 3.5)/2 = 14\text{ kN}\)
Column S/W \(2 \times 12.0 = 24\text{ kN}\)
\[\text{481 kN}\]
Imposed Load
Beams in Y-Dir \(2 \times 53.25 \times (9.7 + 5.0)/2 = 783\text{ kN}\)

The maximum bending moment occurs when BY1 maximum loaded while BY2 minimum loaded at 1/F. The critical situation arises when the axial forces transferred from floors above 1/F are also maximum, as illustrated in the figure on the left. (Other loading cases will be explored in Question B)

**Design Axial Forces**
Reduction to imposed load = \(1 – 2 \times 0.5\% = 0.9\)
Axial load from above 1/F \(1.4 \times 481 + 1.6 \times 783 \times 0.9 = 1801\text{ kN}\)
Beam in Y-Dir at 1/F, BY1  
\[(1.4 \times 30.16 + 1.6 \times 53.25) \times 9.7/2 = 618 \text{ kN}\]

Beam in Y-Dir at 1/F, BY2  
\[1.0 \times 30.16 \times 5.0/2 = 75 \text{ kN}\]

Beams in X-Dir at 1/F  
\[1.4 \times 2.0 \times (3.6 + 3.5)/2 = 10 \text{ kN}\]

Column S/W  
\[1.4 \times 12.0 = 17 \text{ kN}\]

\[2512 \text{ kN}\]

**Design Moment in Y-Direction (i.e. about X-Axis)**

Max Design Load on BY1  
\[1.4 \times 30.16 + 1.6 \times 53.25 = 127.4 \text{ kN/m}\]

Min Design Load: on BY2  
\[1.0 \times 30.16 = 30.2 \text{ kN/m}\]

\[FEM_{B1} = (1/12) \times 127.4 \times 9.7^2 = 999 \text{ kN-m}\]

\[FEM_{B2} = (1/12) \times 30.2 \times 5.0^2 = 63 \text{ kN-m}\]

Unbalanced moment, \(M_{un} = 999 - 63 = 936 \text{ kN-m}\)

**Moment Distribution**

\[K_{cu} = 4 \times (1/12 \times 350 \times 400^3) / 3500 = 2.133 \times 10^6 \text{ mm}^3\]

\[K_{ci} = K_{cu} = 2.133 \times 10^6 \text{ mm}^3\]

\[0.5K_{b1} = 2 \times (1/12 \times 300 \times 700^3) / 9700 = 1.768 \times 10^6 \text{ mm}^3\]

\[0.5K_{b2} = 2 \times (1/12 \times 300 \times 700^3) / 5000 = 3.430 \times 10^6 \text{ mm}^3\]

Moment Distribution Factor = \(2.133 / (2 \times 2.133 + 1.688 + 3.430) = 0.225\)

Therefore,

Design moment to column, \(M = 936 \times 0.225 = 211 \text{ kN-m}\)

**Steel Area by Design Chart**

\[d = 400 - 40 - 10 - 40/2 = 330\]

\[d/h = 330 / 400 = 0.825 \text{ (use 0.80)}\]

\[N/(bh_{cu}) = 2512 \times 10^3 / (350 \times 400 \times 40) = 0.45\]

\[M/(bh_{cu}^2) = 211 \times 10^6 / (350 \times 400^2 \times 40) = 0.094\]

\[A_s f_j/(bh_{cu}) = 0.33\]

\[A_s = 0.33 \times 350 \times 400 \times 40 / 500 = 3696 \text{ mm}^2\]
Question B

For the column in question A, determine the main reinforcement required for the following two loading cases:

(i) Maximum design axial load with coexist moment
(ii) Minimum design axial load from floors above with maximum moment

Summary of Loading (from Question A)

Loading to Beams in Y-Directions

<table>
<thead>
<tr>
<th></th>
<th>Dead Load 30.16 kN/m</th>
<th>Imposed Load 53.25 kN/m</th>
</tr>
</thead>
</table>

Loading to Beams in X-Direction

<table>
<thead>
<tr>
<th></th>
<th>Dead Load 2.02 kN/m</th>
<th>Imposed Load 0.00 kN/m</th>
</tr>
</thead>
</table>

Self-weight of the column

<table>
<thead>
<tr>
<th></th>
<th>Dead Load 12.0 kN per floor</th>
</tr>
</thead>
</table>

Axial Load from the Floors Above 1/F

<table>
<thead>
<tr>
<th></th>
<th>Dead Load 481 kN</th>
<th>Imposed Load 705 kN (reduced by 10%)</th>
</tr>
</thead>
</table>

Solution

Case (i) – Max. Axial Load

Design Axial Forces

\[
A_{x,pr} = 4 \times 1257 = 5028 \text{ mm}^2
\]

\[
\frac{100A_{x}}{(bh)} = 3.6 > 0.8 \quad \text{and} \quad < 6 \quad \text{(Steel Ratio ok)}
\]

Axial load from above 1/F

\[
1.4 \times 481 + 1.6 \times 783 \times 0.9 = 1801 \text{ kN}
\]

Beam in Y-Dir at 1/F, BY1

\[
(1.4 \times 30.16 + 1.6 \times 53.25 \times 0.9) \times 9.7/2 = 577 \text{ kN}
\]

Beam in Y-Dir at 1/F, BY2

\[
(1.4 \times 30.16 + 1.6 \times 53.25 \times 0.9) \times 5.0/2 = 297 \text{ kN}
\]

Beams in X-Dir at 1/F

\[
1.4 \times 2.02 \times (3.6 + 3.5)/2 = 10 \text{ kN}
\]

Column S/W

\[
1.4 \times 12.0 = 17 \text{ kN}
\]

\[
2702 \text{ kN}
\]

Design Moment in Y-Direction (i.e. about X-Axis)

Max Design Load:

\[
1.4 \times 30.16 + 1.6 \times 53.25 = 127.4 \text{ kN/m}
\]

Min Design Load:

\[
1.0 \times 30.16 = 30.2 \text{ kN/m}
\]

\[
FEM_{B1} = \frac{1}{12} \times 127.4 \times 9.7^2 = 999 \text{ kN-m}
\]

\[
FEM_{B2} = \frac{1}{12} \times 127.4 \times 5.0^2
\]
Unbalance moment, $M_{es} = 999 - 265 = 734 \text{ kN-m}$

Moment Distribution Factor = 0.225

Therefore,

Design moment to column, $M = 734 \times 0.225 = 165 \text{ kN-m}$

Steel Area by Design Chart

\[
\frac{d}{h} = \frac{330}{400} = 0.825 \text{ (use 0.80)}
\]

\[
\frac{N}{bh f_{cu}} = \frac{2702 \times 10^3}{350 \times 400 \times 40} = 0.48
\]

\[
\frac{M}{bh^2 f_{cu}} = \frac{165 \times 10^6}{350 \times 400^2 \times 40} = 0.074
\]

\[
\frac{A_s f_y}{bh f_{cu}} = 0.30 \text{ (Not critical compared with Q.A)}
\]

**Case (ii) – Min. Axial Load**

**Design Axial Forces**

Axial load from above 1/F

\[
1.0 \times 481 = 481 \text{ kN}
\]

Beam in Y-Dir at 1/F, BY1

\[
(1.4 \times 30.16 + 1.6 \times 53.25 \times 0.9) \times 9.7/2 = 577 \text{ kN}
\]

Beam in Y-Dir at 1/F, BY2

\[
1.0 \times 30.16 \times 5.0/2 = 75 \text{ kN}
\]

Beams in X-Dir at 1/F

\[
1.0 \times 2.02 \times (3.6 + 3.5)/2 = 10 \text{ kN}
\]

Column S/W

\[
1.0 \times 12.0 = 12 \text{ kN}
\]

\[\text{Total} = 1155 \text{ kN}\]

**Design Moment in Y-Direction (i.e. about X-Axis)**

Max Design Load:

\[
1.4 \times 30.16 + 1.6 \times 53.25 = 127.4 \text{ kN/m}
\]

Min Design Load:

\[
1.0 \times 30.16 = 30.2 \text{ kN/m}
\]

\[
FEM_{B1} = \frac{1}{12} \times 127.4 \times 9.7^2 = 999 \text{ kN-m}
\]

\[
FEM_{B2} = \frac{1}{12} \times 30.2 \times 5.0^2 = 63 \text{ kN-m}
\]

Unbalance moment, $M_{es} = 999 - 63 = 936 \text{ kN-m}$

Moment Distribution Factor = 0.225

Therefore,

Design moment to column, $M = 936 \times 0.225 = 211 \text{ kN-m}$

Steel Area by Design Chart

\[
\frac{d}{h} = \frac{330}{400} = 0.825 \text{ (use 0.80)}
\]
The results of the three loading cases in Questions A, B(i) and B(ii) are plotted in the following figure.

For this column, it is apparent that the critical load case is "A", which is well above the balanced failure mode. However, in some cases, if the result for maximum design axial load is close to balanced failure mode, the other cases may become more critical.
6.2.5 Design for Biaxial Moment

When a symmetrically-reinforced rectangular column section is subjected to bending in two directions as shown in figure on the right, it can be designed by transforming the biaxial moment into uniaxial moment using the following equations (Eqn 6.57 and 6.58 of HKCP-2013), and the section can then be designed for an increased moment about the corresponding axis only.

For $M_x/h' \geq M_y/b'$, $M_x' = M_x + \beta(h'/b')M_y$

\[6.5\]

For $M_x/h' < M_y/b'$, $M_y' = M_y + \beta(b'/h')M_x$

\[6.6\]

The values of $\beta$ are given by the following table.

<table>
<thead>
<tr>
<th>$N/(bh_{fcu})$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>$\geq 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.00</td>
<td>0.88</td>
<td>0.77</td>
<td>0.65</td>
<td>0.53</td>
<td>0.42</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 6.2 – Values of the Coefficient $\beta$ for Biaxial Bending

(Table 6.14 of HKCP-2013)

6.2.6 Example – Design of Short Braced Column under Biaxial Loads

**Question**

Determine the main reinforcement for the column section shown in the figure on the right hand side with the following design parameters.
Design Parameters

Design Axial Load, N = 1800 kN
Design Moment about X-axis, M_x = 85 kN-m
Design Moment about Y-axis, M_y = 95 kN-m
\( f_{cu} = 30 \text{ MPa} \)
\( f_y = 500 \text{ MPa} \)
Cover = 40 mm
Preferred link size = 10
Preferred bar size = 32

Solution

\[ b = 350 \text{ mm} \]
\[ b' = 350 - 40 - 10 - 32/2 = 284 \text{ mm} \]
\[ h = 400 \text{ mm} \]
\[ h' = 400 - 40 - 10 - 32/2 = 334 \text{ mm} \]
\[ M_x / h' = 85 / 334 = 0.25 \]
\[ M_y / b' = 95 / 284 = 0.33 \]

As \( M_y / b' > M_x / h' \), increase \( M_y \) for design

\[ N/(bh f_{cu}) = 1800 \times 10^3 / (350 \times 400 \times 30) = 0.43 \]
\[ \beta = 0.53 - (0.53-0.42) \times 0.3 = 0.497 \]
\[ M_y' = 95 + 0.497 \times (284/334) \times 85 = 131 \text{ kN-m} \]

As bending about Y-axis is considered,

the following parameters are used for Design Chart

\[ h = 350 \]
\[ d = 284 \]
\[ d/h = 284/350 = 0.81 \text{ (use 0.80)} \]
\[ N/(bh f_{cu}) = 0.43 \]
\[ M/(bh^2 f_{cu}) = 131 \times 10^6 / (400 \times 350^2 \times 30) = 0.089 \]
\[ A_s f_y/(bh f_{cu}) = 0.29 \]
\[ A_s = 0.29 \times 400 \times 350 \times 30 / 500 = 2436 \text{ mm}^2 \]

(Provide 4 T32)

\[ A_{s,prov} = 4 \times 804 = 3216 \text{ mm}^2 \]
\[ 100A_s/(bh) = 2.3 > 0.8 \text{ and } < 6 \]
(Steel Ratio ok)

The definition of \( h \) is the dimension of the column section in the direction of the design moment.
6.2.7 Transverse Reinforcement

In addition to the main reinforcement bars provided in the longitudinal direction along the height of column, transverse reinforcement, in the forms of links, cross-ties, etc., should be provided to restrain the main bars from buckling under compression. The requirements on the provision of these transverse reinforcements are:

i. Bar size $\geq 6$ or $1/4 \times$ size of the longitudinal bar.

ii. Spacing $\leq 12 \times$ size of the longitudinal bar, b and h, 400mm.

iii. All corner bars, alternate bars shall be restrained in 2 directions.

iv. No unrestrained bars should be more than 150mm from a restrained bar.

v. 2-direction restraint should be provided by links passing round the bar with an included angle of not more than $135^\circ$.

vi. Circular or spiral links should be provided to circular column.

The following figures extracted from Figure 9.5 of Cl.9.5.2 of HKCP-2013 shows the details of the requirements.
6.3 Walls

A vertical member is regarded as wall instead of column when the length to thickness ratio of the section is more than 4.

With its high in-plane stiffness, walls are usually used in building structure to resist lateral forces acting on the building by in-plane bending of the wall. The design of wall for the combined effect of axial load and in-plane moment is quite similar to that for column. A wall can be visualized as strips of column placed together side-by-side for design.

Walls may also be designed to resist out-of-plane moment, e.g. walls of water tank, earth retaining structures, etc. of which, if the design force is predominantly transverse bending, the design principles of slab can be adopted.

6.3.1 Classification of Walls

(a) Slender or Stocky Wall

Similar to column, walls are categorized into braced or non-braced, and then, according to the effective height to thickness ratio, classified into slender or stocky wall. For wall, the out-of-plane direction of bending governs the classification. If a wall is classified as slender, additional out-of-plane moment has to be designed for.\(^5\) This chapter focuses on the design for stocky braced reinforced wall\(^6\), of which

\[
\frac{L_o}{h} < 15
\]

---

\(^5\) For slender reinforced concrete wall, determination of additional moment is similar to that for slender column. Refer to Cl. 6.2.1.3 of HKCP-2013 for details. It is beyond the scope of this chapter.

\(^6\) Take note that the determination of \(L_o\) and the limits on \(L_o/h\) are different for non-braced wall and for plain wall. Refer to the design code for details.
Example
A 180mm thick reinforced wall is constructed monolithically with 150mm slabs at each floor and the floor-to-floor height is 3200mm, and the building structure is braced.

Unrestrained height, \( L_0 = 3200 - 150 = 3050 \text{mm} \).
As the slab thickness at both top and bottom is thinner than the wall, the restraints belong to condition 2, and therefore \( \beta = 0.85 \).
Effective height \( L_e = 0.85 \times 3050 = 2593 \text{mm} \).
\( L_e/h = 2593/180 = 14.4 < 15 \). It is a stocky wall.

(b) Reinforced or Plain Wall

When the design forces of the wall is so small that the strength of the concrete alone is adequate to resist the design forces, the wall can then be designed as plain wall. In other words, plain wall is walls of which the design is based on without reinforcement, although nominal reinforcement of 0.25% has to be provided. On the other hand, reinforced wall are walls of which reinforcement bars have to be provided to resist the design forces, and the nominal reinforcement for reinforced wall is 0.4%.

6.3.2 Design for Axially Loaded Stocky Wall

If a stocky braced wall supports an approximately symmetrical arrangement of slabs, it can be designed by the following formula (Eqn 6.59 of HKCP-2013):

\[
 n_w \leq 0.35f_{cu}A_c + 0.67f_yA_{sc} \tag{6.7}
\]

where, \( n_w \) is the total design axial load on the wall. An allowance for \( \gamma_m \) is included in the formula.

---

7 Not only the nominal steel requirement is different, the determination of effective height, the design formulae, etc. for plain wall are also different from those for reinforced wall. Hence, you have to decide at the outset whether the wall is to be designed as plain wall or reinforced wall, or you can do it by try and error. This chapter focusses on the design of reinforced wall.
It is identical to [6.4] above for column design, which deems to have taken into account of the out-of-plane bending moment induced by asymmetric loading of the slabs.

### 6.3.3 Design for Axial Load and Out-of-plane Moment

Where a wall section is designed for combined axial load and out-of-plane moment, a unit length, i.e. 1.0m, of the wall can be designed as a column and the Column Design Charts can be used for design.

**Example**

A 3000 x 250 braced stocky wall is subjected to a design axial load of 4000 kN/m and an out-of-plane design moment of 145 kN-m/m. Given $f_y = 500$ MPa, $f_{cu} = 40$ MPa, cover = 30mm.

\[
\frac{N}{bh f_{cu}} = 0.40 \\
\frac{M}{bh^2 f_{cu}} = 0.058 \\
d = 250 - 30 - 10 - 16/2 = 202\text{mm}, \quad d/h = 0.81 \quad (\text{use } 0.80) \\
\text{From Column Design Chart, } A_s f_y/(bh f_{cu}) = 0.13 \\
\text{Therefore, } A_s = 0.13 \times 250 \times 1000 \times 40/500 = 2600 \text{ mm}^2/\text{m} \\
\text{Provide T16-150 on both faces. } A_{s,\text{prov}} = 2680 \text{ mm}^2/\text{m}
\]

**Note:** In this example, loads are given in per meter run, and therefore the calculations is also done in per meter run without taking into account of the overall length of the wall.

### 6.3.4 Design for Axial Load and In-plane Moment

There are various methods to analysis and design wall section subjected to in-plane moment. Two simplified methods are introduced here.

(a) M-N Interaction Chart Method
Similar to column, assuming an amount of reinforcement for the wall section, try various values of the depth of neutral axis, \( x \), to obtain an M-N interaction curve. If the point of coordinates of the design forces, \((M, N)\) falls within and very close to the curve, the amount of reinforcement you have assumed is therefore acceptable; otherwise, the amount of reinforcement has to be adjusted to derive another M-N curve until an appropriate one is obtained.\(^8\)

For simplicity, Column Design Chart can be used if the wall is symmetrically reinforced, as described below:

(i) Treat the wall as a rectangular column section with its length\(^9\) as depth, \( h \), while its thickness as breath, \( b \).

(ii) Group the reinforcement bars into top half and bottom half. Determine the depth, \( d \), to the centroid of the bottom steel bars. Use this value to obtain the \( d/h \) ratio. If the wall \textit{uniformly reinforced} along its length, \( d = h - h/4 = 0.75h \), and therefore \( d/h \) is simply taken as 0.75.

(iii) Calculate \( N/(bh_{cu}) \) and \( M/(bh^2_{cu}) \) and then obtain the value of \( A_s f_y/(bh_{cu}) \) from the Column Design Chart with corresponding \( d/h \) value.

\( \text{(b) Linear Load Distribution Method} \)

The combined effect of axial load and in-plane moment can be regarded as a linear distribution of axial forces and then design the wall as strips of column as described below:

---

\(^8\) Pay attention that this method is based on the assumption that plane section remain plane, i.e. strain distribution is linear. However, if the height to length ratio of the wall is small, it will behave more like a deep beam.

\(^9\) Be careful with the usage of the notation, \( h \), which is not referring to the length of the wall, but, with its usual meaning in slab and beam design, to which thickness and depth is referred, and is therefore very often referring the thickness of the wall. However, when the length of the wall is designed for in-plane bending, \( h \) refers to the length. To be more precise, \( h \) is defined as the dimensions of the section in the direction of the design moment.
(i) Transform the design moment, $M$, and the design axial load, $N$, by elastic method into a linear distribution of axial force per unit length along the length of the wall as shown below.

(ii) Visualize the wall as $n$ number of columns with length equal to $b/n$ each. Calculate the design axial load at the center of each strip by linear interpolation.

(iii) Determine the amount of steel required for each strip of column by [6.3] above for column under axial load.

(vi) If the value of the design axial load of a strip is negative, that means it is under tension; ignore the concrete and provide reinforcement to take up the tension.

### 6.3.5 Examples

**Question A**
Determine the main reinforcement for the following stocky braced wall by using appropriate Column Design Chart.

**Design Parameters**
- Wall dimensions $b \times h = 350 \times 4000$
- Design Axial Load, $N = 27000$ kN
Design in-plane Moment, \( M = 12000 \text{ kN-m} \)

\[
f_{cu} = 40 \text{ MPa} \\
f_y = 500 \text{ MPa} \\
\text{Cover} = 30 \text{ mm} \\
\text{Preferred link size} = 10 \\
\text{Preferred bar size} = 32
\]

**Solution**

\[
\frac{N}{(bh)f_{cu}} = \frac{27000 \times 10^3}{(350 \times 4000 \times 40)} = 0.48
\]

\[
\frac{M}{(bh^2)f_{cu}} = \frac{12000 \times 10^6}{(350 \times 4000^2 \times 40)} = 0.0536
\]

Assume the steel is uniformly distributed, therefore \( d/h = 0.75 \)

From Column Design Chart of \( d/h = 0.75 \)

\[A_s f_y / (bh f_{cu}) = 0.22\]

\[A_s = 0.22 \times 350 \times 1000 \times 40 / 500\]

\[= 6160 \text{ mm}^2 \text{ per meter}\]

Limits to steel ratio for column refer to Table 2.1 in Chapter 2 or the Annex.

\[A_{s, prov} = 2 \times 804 / 0.250 = 6432 \text{ mm}^2\]

\[100A_s / (bh) = 1.8 > 0.4 \text{ and } < 4 \text{ (Steel Ratio ok)}\]

**Question B**

Determine the main reinforcement for the following stocky braced wall by using Linear Load Distribution Method.

**Design Parameters**

Wall dimensions \( h \times b = 350 \times 4000 \)

Design Axial Load, \( N = 27000 \text{ kN} \)

Design Moment in-plane \( M = 12000 \text{ kN-m} \)

\[f_{cu} = 40 \text{ MPa} \]

\[f_y = 500 \text{ MPa} \]

Cover = 30 mm

Preferred link size = 10

Preferred bar size = 32

**Solution**

\[
N/b = 27000 / 4 = 6750 \text{ kN/m}
\]

\[
6M/b^2 = 6 \times 12000 / 4^2 = 4500 \text{ kN/m}
\]
As $6750 - 4500 > 0$, there is no tension.

Divide the wall into 4 strips

Width of each strip = $4000 / 4 = 1000$ mm

The max load on each strip is tabulated as follows

<table>
<thead>
<tr>
<th>Strip</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Loading due to N</td>
<td>6750</td>
<td>6750</td>
<td>6750</td>
<td>6750</td>
</tr>
<tr>
<td>Loading due to M</td>
<td>-3375</td>
<td>-1125</td>
<td>1125</td>
<td>3375</td>
</tr>
<tr>
<td>Total Axial Force</td>
<td>3375</td>
<td>5625</td>
<td>7875</td>
<td>10125</td>
</tr>
<tr>
<td>Capacity of Concrete, $0.4f_{cu}bh$</td>
<td>5600</td>
<td>5600</td>
<td>5600</td>
<td>5600</td>
</tr>
<tr>
<td>Force taken up by steel</td>
<td>0</td>
<td>25</td>
<td>2275</td>
<td>4525</td>
</tr>
<tr>
<td>$A_s = N_s/(0.75f_y)$</td>
<td>1400</td>
<td>1400</td>
<td>6067</td>
<td>12067</td>
</tr>
<tr>
<td>Provide</td>
<td>T16-275</td>
<td>T16-275</td>
<td>T32-250</td>
<td>T32-125</td>
</tr>
<tr>
<td>$100A_s/(bh)$</td>
<td>0.42</td>
<td>0.42</td>
<td>1.84</td>
<td>3.68</td>
</tr>
</tbody>
</table>

**Comments:**

1. The nominal reinforcement for wall is 0.4%. Therefore, the amount of steel required to strip 1 is $350 \times 1000 \times 0.4 / 100 = 1400$ mm$^2$.

2. Although the configuration and design forces for the walls in questions A and B are the same, two design methods come up with two different reinforcement requirements.

3. The total amount of steel required in Question A is $4 \times 6160 = 24640$ mm$^2$, while that for Question B is $1400 + 1400 + 6067 + 12067 = 20934$ mm$^2$, about 15% lesser.

4. The reinforcement distribution in Question A is uniform, and therefore, it does not matter if the direction of design moment is reversed. On the other hand, the distribution of reinforcement in Question B is better positioned to take up the stress induced by the specific direction of the moment.

5. If the design in Question B has to cater for reversal of bending, the amount of steel required will then be $2 \times (6067 + 12067) = 36268$ mm$^2$, about 47% more than that of Question A. However, it has to note that the equation used to determine steel area in Question B has taken into account of the out-of-plane moment due to nominal eccentricity of the axial load, which has not been accounted for in Question A.
6.3.6 Notes on Detailing

The design code limits the spacing of vertical bars in wall (Cl.9.6.2 of HKCP-2013) as follows:

Spacing of vertical bars < lesser of 400mm and 3 x wall thickness

If the wall is subjected to out-of-plane bending, bar spacing requirements similar to that for slab may have to be adopted.

In addition, vertical compression bars have to be restrained from buckling by provision of horizontal bars or transverse bars. Details refer to the design code (Cl.9.6.3 and Cl.9.6.4 of HKCP-203).\(^\text{10}\)

\(^{10}\) The rules of detailing is beyond the scope of this chapter. Refer to the design code for details.
Notes on the Derivation of the Column Design Charts

The charts are derived by using the simplified rectangular stress block for symmetrically-reinforced rectangular section subject to uniaxial bending moment as shown below:

Abbreviations:
\( h, b \) : overall depth and breadth of the rectangular section
\( d \) : depth from the top face to the centroid of the steel in the bottom face
\( d' \) : depth from the top face to the centroid of the steel in the top face, \( d' = h - d \)
\( N, M \) : Axial force and moment
\( f_{cu}, f_{cm} \) : concrete cube strength and ultimate compressive strain of concrete
\( k_0 \) : effective compressive strength of concrete in the section
\( k_0' \) : depth of the flexural compressive stress block in the concrete
\( k \) : ratio of \( x_h \), where \( x \) is the depth to neutral axis, \( x = k h \)
\( \varepsilon_{st}, f_{st} \) : strain and stress of steel bars in bottom face
\( \varepsilon_{st}, f_{st} \) : strain and stress of steel bars in top face
\( E_s, f_y \) : Elastic Modulus and yield stress of steel
\( \gamma_m \) : Material factor of safety for steel bars

Dimensionless Parameters
\( [A_s] = A_s/(bh\gamma_m) \)
\( [M] = M/(bh^2\gamma_m) \)

\[ [N] = M/(bh\gamma_m) \]
\[ [f_y] = f_{y\gamma_m} \]
\[ [f_t] = f_{t\gamma_m} \]

Sign convention
Compression: +ve
Tension: -ve

Compatibility of strain
Strain of steel bars at bottom face:
\[ \varepsilon_{st} = \varepsilon_{cm} (1 - d'/h) / k \]
Therefore, stress in the steel bar:
\[ f_{st} = E_s \varepsilon_{st} (k - d'h) / k \] and \( f_{st} \gamma_m < f_y < f_{t\gamma_m} \) \hspace{1cm} \text{Eq. (a)}

or \[ [f_{st}] = \varepsilon_{cm} (k - d'h) / (k'f_y\gamma_m) \] and \( -1 < \gamma < [f_{st}] < 1 \) \hspace{1cm} \text{Eq.1a}

Strain of steel bars at top face:
\[ \varepsilon_{st} = \varepsilon_{cm} (k + d'h) / k \]
Therefore, stress in the steel bar:
\[ f_{st} = E_s \varepsilon_{st} (k + d'h) / k \] and \( f_{st} \gamma_m < f_y < f_{t\gamma_m} \) \hspace{1cm} \text{Eq. (b)}

or \[ [f_{st}] = \varepsilon_{cm} (k + d'h) / (k'f_y\gamma_m) \] and \( -1 < \gamma < [f_{st}] < 1 \) \hspace{1cm} \text{Eq.1b}

Case 1
The whole section is under compression and both top and bottom steel have yielded under compression.

When \( f_{st} = f_{t\gamma_m} \):

\[ \varepsilon_{cm} (k - d'h) / k = f_{t\gamma_m} \]

\[ k = (d'h)/(1 - [f_{st}]\gamma_m) \] \hspace{1cm} \text{Eq. (c)}

When \( k > \) the value given by Eq (c):

\[ N = k\varepsilon_{cm}\varepsilon_{cm} (f_{t\gamma_m})A_s \]
Case 1

When the compression seal is in the top face, the neutral axis is located at the top face. If $h < h_i$, then

\[ k = \left( \frac{h}{h_i} \right)^n \]

or

\[ N = kN' \]

At the section, the compressive stresses are

\[ \sigma_c = -\frac{kN}{A} \]

\[ \sigma_s = \frac{kN}{A} \]

Case 2

When the neutral axis is exactly on the steel bar at the top face, i.e., $h = h_i$, then

\[ k = 1 \]

or

\[ N = N' \]

At the section, the compressive stresses are

\[ \sigma_c = 0 \]

\[ \sigma_s = 0 \]

Case 3

When the neutral axis is located in the interior of the cross-section, i.e., $h > h_i$, then

\[ k = \left( \frac{h}{h_i} \right)^m \]

or

\[ N = kN' \]

At the section, the compressive stresses are

\[ \sigma_c = -\frac{kN}{A} \]

\[ \sigma_s = \frac{kN}{A} \]

Remarks:

1. The value of $k$ and $N' = N/A$ are used in the analysis.
2. The neutral axis is located at the top face of the section.
3. The compressive stress at the neutral axis is zero.
4. The tensile stress at the neutral axis is zero.
5. The compressive stress at the bottom face is zero.
6. The tensile stress at the bottom face is zero.
7. The compressive stress at the top face is zero.
8. The tensile stress at the top face is zero.
9. The compressive stress at the top face is zero.
10. The tensile stress at the top face is zero.

For ACI 437-05, the following equations are adopted:

\[ \sigma_c = -\frac{kN}{A} \]

\[ \sigma_s = \frac{kN}{A} \]

\[ k = \left( \frac{h}{h_i} \right)^m \]

\[ N = kN' \]

\[ M = M' \]

\[ \theta = \theta' \]

\[ f_y = f_y' \]

\[ f_c = f_c' \]

Where:

- $k$ is the reduction factor.
- $N'$ is the factored compressive force.
- $M'$ is the factored bending moment.
- $\theta'$ is the factored rotation.
- $f_y'$ is the yield strength of the steel.
- $f_c'$ is the compressive strength of the concrete.
## Part Plan of a Typical Floor

**Imposed Load:** 15.0 kPa

**Allowable for Finish:** 70 kPa

### Section

**Notations:**
1. Beam slips is presented in (DEPTH X DIA (MM))
2. Grade of Concrete: C40
3. Steel for main bar: Grade 500
4. Steel for link: Grade 250

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<th><strong>Dwg. Title</strong></th>
<th><strong>Dwg. No.</strong></th>
<th><strong>Rev.</strong></th>
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<td>IWG-01b</td>
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### Part Plan of a Typical Floor

- **Dimensions:**
  - 9900 x 5000
  - 3000 x 3500
  - 2500 x 2500

- **Beams:**
  - Beam 1 (400x450)
  - Beam 2 (400x450)
  - Beam 3 (400x450)
  - Beam 4 (400x450)

- **Columns:**
  - Column 1 (400x450)
  - Column 2 (400x450)
  - Column 3 (400x450)

- **Notations:**
  - (D1) x (D2)
  - (L1) x (L2)

### Section

- **Levels:**
  - G/F
  - 1/F
  - 2/F
  - 3/F

- **Dimensions:**
  - 3500
  - 3000
Self-Assessment Questions

Q.1 The following are the dimensions for a column and the connecting beams of a building structure which is provided with shear walls to resist lateral loads.

Column: \( h = 550 \text{ mm (in Y-direction)} \)
\( b = 450 \text{ mm} \)
Floor-to-floor height = 6800 mm

Beams at the Top
- Depth of the beam in X-direction = 400 mm
- Depth of the beam in Y-direction = 450 mm

Beam at the Bottom
- Depth of the beam in X-direction = 400 mm
- Depth of the beam in Y-direction = 450 mm

(a) Determine the effective heights, \( L_{ex} \) and \( L_{ey} \).
(b) Determine the effective height to width ratios.
(c) Classify whether the column is short or not.

Q.2 A short braced column at the roof floor of a building structure supporting two beams, B1 and B2, on each side in the Y-direction with the following design parameters.

Column dimensions, \( h = b = 400 \text{ mm} \)
Floor-to-roof height, \( L_c = 4000 \text{ mm} \)
Beam dimensions, \( h \times b = 650 \times 300 \text{ mm} \)
- Span of B1, \( L_{b1} = 9000 \text{ mm} \)
- Span of B2, \( L_{b2} = 9000 \text{ mm} \)

Loading on the beams B1 & B2
- Dead Load, \( g_k = 100 \text{ kN/m (beam self-weight included)} \)
- Imposed Load, \( q_k = 75 \text{ kN/m} \)
- Loading in X-direction: (negligible,)

\( f_{cu} = 40 \text{ MPa} \)
\( f_{y} = 500 \text{ MPa} \)
Cover = 40 mm
Preferred link size = 10
Preferred bar size = 40

(a) Determine the maximum design moment transmitted from the beams by Simplified Sub-frame Analysis.
(b) Determine the co-existing design axial load for the design moment in (a).
(c) Find the steel area required for the above design forces by using Column Design Chart.
(d) Determine the maximum design axial force for the column by tributary area method.
(e) Use equation 6.4 (i.e. Eqn.6.56 of HKCP-2013) to find the steel area required.
(f) Compare the results in (c) and (e).

Q.3 Given the following information of a short braced column which is subjected to axial load only:

Column dimensions: h x b = 400 x 400 mm
Design Axial Compression, N = 4000 kN
\( f_{cu} = 40 \text{ MPa} \)
\( f_y = 500 \text{ MPa} \)
Cover = 40 mm
Preferred link size = 10
Preferred bar size = 40

(a) Determine the nominal design moment of the column.
(b) Determine the \( A_s f_y/(bh_{cu}) \) value and the \( A_s \) required for the column from the Column Design Chart.
(c) Determine the area of steel required by using Eqn 6.55 of HKCP-2013.
(d) Compare the results of (b) and (c)

Q.4 For the column C2 in Question A of 6.2.4, if the length of the column from G/F to 1/F is increased to 4.5m and the others remain unchanged,

(a) Determine design moment to the column.
(b) Determine the steel area required.
Q.5 Determine the area of main reinforcement required for the following braced short columns by completing the table.

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Axial Load, $N$ =</td>
<td>4800 kN</td>
<td></td>
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<tr>
<td>Preferred link size =</td>
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<tr>
<td>Design Moment about X-axis, $M_x$ =</td>
<td>450 kN-m</td>
<td></td>
<td></td>
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<tr>
<td>Preferred bar size =</td>
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<tr>
<td>Design Moment about Y-axis, $M_y$ =</td>
<td>200 kN-m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cover =</td>
<td>40 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_y$ =</td>
<td>500 MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{cu}$ =</td>
<td>40 MPa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                  |      |      |      |
| $b$ = | 500 | 370 | 670 |
| $h$ = | 500 | 670 | 370 |
| $b'$=    |      |      |      |
| $h'$=    |      |      |      |
| $M_x / h'$ = |      |      |      |
| $M_y / b'$ = |      |      |      |
| $N/(bh_{cu})$ = |      |      |      |
| $\beta = $ |      |      |      |
| $M =$ |      |      |      |
| $d/h =$ |      |      |      |
| $M/(bh^2_{cu}) = $ |      |      |      |
| $A_s f_y/(bh_{cu}) =$ |      |      |      |
| $A_s =$ |      |      |      |

Answers:
Q1a: $L_{sx} = 5398$, $L_{sy} = 5440$; Q1b: $L_{sw}/h = 9.8$, $L_{sw}/b = 12.1$; Q1c: Short Column
Q2a: $M_{sx} = 1080$kN-m; $M = 444$kN-m; Q2b: $M_{sx} = 1642$kN;
Q2c: $N/(bh_{cu}) = 0.26$, $M/(bh^2_{cu}) = 0.174$, use Chart $(d/h=0.80)$, $A_s f_y/(bh_{cu}) = 0.50$, $A_s = 6400$mm$^2$;
Q2d: $2340+22(SW) = 2362$kN; Q2e: $364$mm$^2$.
Q3a: $80$kN-m; Q3b: $0.30$, $3840$mm$^2$; Q3c: $3840$mm$^2$
Q4a: $173$kN-m; Q4b: $A_s f_y/(bh_{cu}) = 0.26$. $A_s = 2912$mm$^2$
Q5a: $M = 538$, $d/h = 0.86$, $A_s = 7600$; Q5b: $M = 627$, $d/h = 0.90$, $A_s = 6200$; $M = 494$, $d/h = 0.81$, $A_s = 10800$
**Tutorial Questions**

*(Present your calculations with detailed working steps in a logical, neat and tidy manner.)*

AQ1 Design the edge column C1 shown on DWG-06 with the following information given.

**Design parameters**

**Column C1**
- Column breadth, \( b = 350 \text{ mm (in X-dir)} \)
- Column depth, \( h = 400 \text{ mm} \)
- Floor-to-floor height = 3500 mm (same for all floors)

**Beams in Y-direction**
- Beams BY1, \( h \times b = 700 \times 300 \text{ mm} \)
- Beam BY1 span, \( L_{by1} = 9900 - 400/2 = 9700 \text{ mm} \)

**Beams in X-direction**
- Beams BX3 & BX4, \( h \times b = 500 \times 300 \text{ mm} \)
- Beam BX3 span, \( L_{bx1} = 3600 \text{ mm} \)
- Beam BX4 span, \( L_{bx2} = 3500 \text{ mm} \)

- Slab thickness = 225 mm (same for all floors)
- Allowance for finishes = 2.0 kPa (same for all floors)
- Imposed load = 15.0 kPa (same for all floors)
- External wall supported on BX3 & BX4 = 10.35 kN/m
- No of storeys = 3
  - \( f_{cu} = 40 \text{ MPa} \)
  - \( f_{y} = 500 \text{ MPa} \)
  - Cover = 40 mm
  - Preferred link size = 10
  - Preferred bar size = 40
The dimensions of a braced reinforced concrete column of a multi-story building are given below and in the figure:

Depth, \( h \) = 550mm  
\( h' \) = 470mm  
Breadth, \( b \) = 400mm  
\( b' \) = 320mm

Clear height between floor beams,  
\( L_o \) = 5,000mm

Concrete: Grade C35  
Reinforcement: Grade 500

The top and bottom ends of the column are connected monolithically to reinforced concrete beams deeper than the dimensions of the column in both directions.

(a) Check the slenderness of the column and comment on its implications on the column design.

(b) The design ultimate loads for the column at roof floor are found to be as follows:
   - Axial compression, \( N = 1,000 \text{ kN} \)
   - Moment about x-axis, \( M_x = 300 \text{ kN-m} \)
   - Moment about y-axis, \( M_y = 100 \text{ kN-m} \)
   Calculate the percentage of steel required for the column at roof floor.

(c) The design ultimate loads for the column at the fourth floor below the roof floor are found to be as follows:
   - Axial compression, \( N = 5,500 \text{ kN} \)
   - Moments are same as (b).
   Calculate the percentage of steel required for the column at this floor.

(d) Estimate the percentage of steel required, by using Eqn 6.56 of HKCP-2013 given below for the column in (c), i.e. \( N = 5,500 \text{ kN} \), and ignoring the design moments.
   \[ N = 0.35f_{cu}A_c + 0.67f_yA_{sc} \]

(e) Discuss your observations in the above calculations.

(This question is adapted from 2012/13 Sem 3 examination paper.)
AQ3 The dimensions of a short braced reinforced concrete column are given in Figure Q.3a on the right hand side:

- Depth, \( h \) = 500 mm
- \( h' \) = 425 mm
- Breadth, \( b \) = 400 mm
- Floor-to-floor height, \( L_c \) = 5000 mm

Concrete: Grade C35
Reinforcement: Grade 500

This column supports two roof beams BY1 and BY2 on each side in the Y-direction with the following information:

- Beam depth = 650 mm
- Beam breadth = 300 mm
- Span of BY1, \( L_{b1} \) = 9000 mm
- Span of BY2, \( L_{b2} \) = 7700 mm

Loading on the beams, BY1 and BY2:
- Dead load, \( g_k \) = 120 kN/m
- Imposed load, \( q_k \) = 80 kN/m

Loading from beams in the X-direction and the self-weight of column can be ignored.

(a) Apply the Simplified Sub-frame Method as illustrated in Figure Q.3b to verify that the maximum design moment transferred from the beams is \( M_x = 705 \) kN-m.

(b) Verify that the design axial load which co-exists with the design moment in (a) is \( N = 1794 \) kN.

(c) Find the steel area of reinforcement required for the design forces in (a) and (b) by using the Column Design Chart and then design the reinforcement for it.

(This question is adapted from 2013/14 Sem 3 examination paper.)