5 ENERGY EQUATION OF FLUID MOTION

5.1 Eulerian Approach & Control Volume

In order to develop the equations that describe a flow, it is assumed that fluids are subject to certain fundamental laws of physics. The pertinent laws are:

1. Conservation of mass;
2. Conservation of energy;

These principles were initially developed for the case of a solid body and the application of these laws to a solid body is relatively straightforward since the body will be of measurable size and mass. However, it is not for a flowing fluid.

To describe the displacement, velocity and acceleration of a fluid, control volume approach (Eulerian approach) is commonly used.

A control volume is a purely imaginary region within a body of flowing fluid. The region is usually at a fixed location and of fixed size. Inside the region, all of the dynamic forces cancel each other. Attention may therefore be focused on the forces acting externally on the control volume. The control volume may be of any shape. Therefore, a shape may be selected which is most convenient for any particular application.

5.2 Continuity Equation

The principle of conservation of mass can be applied to a flowing fluid. Considering any fixed region in the flow constituting a control volume,

\[
\text{Mass of fluid entering per unit time} = \text{Mass of fluid leaving per unit time} + \text{Increase of mass of fluid in the control volume per unit time}
\]
Fluid Mechanics  Chapter 5 – Energy Equation

Continuity of Flow

For steady flow, the mass of fluid in the control volume remains constant and the relation reduces to

\[
\text{Mass of fluid entering per unit time} = \text{Mass of fluid leaving per unit time}
\]

Apply this principle to steady flow in a streamtube as shown below,

Continuous flow through a streamtube

If there is no flow being evacuated from the stream tube except at the outlet section 2, then

\[
Q = \rho V_1 A_1 = \rho V_2 A_2
\]

\[
\therefore Q = V_1 A_1 = V_2 A_2
\]

where \( V_1, V_2 \) are the mean velocities at sections 1 and 2 respectively.

The continuity equation for liquids many be generally expressed in the form

\[
Q = V \times A = \text{constant}
\]
where \( V \) and \( A \) are velocity of flow and the cross-sectional area normal to the flow respectively at any section of the liquid stream.

The continuity equation can also be applied to determine the relation between the flows into and out of a junction. For steady condition,

\[
\text{Total inflow to junction} = \text{total outflow from junction}
\]

\[
\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3
\]

For an incompressible flow, \( \rho_1 = \rho_2 = \rho_3 \), then

\[
Q_1 = Q_2 + Q_3
\]

or

\[
A_1 V_1 = A_2 V_2 + A_3 V_3
\]

In general, if we consider flow towards the junction as positive and flow away for the junction as negative, then for steady flow at any junction the algebraic sum of all the mass flows must be zero:

\[
\Sigma \rho Q = 0
\]
**Worked example:**

Water flows through a pipe AB of diameter $d_1 = 50$ mm, which is in series with a pipe BC of diameter $d_2 = 75$ mm in which the mean velocity $V_2 = 2$ m/s. At C the pipe forks and one branch CD is of diameter $d_3$ such that the mean velocity $V_3$ is 1.5 m/s. The other branch CE is of diameter $d_4 = 30$ mm and conditions are such that the discharges $Q_2$ from BC divides so that $Q_4 = \frac{1}{2} Q_3$. Calculate the values of $Q_1$, $V_1$, $Q_2$, $Q_3$, $d_3$, $Q_4$ and $V_4$.

![Diagram of fluid flow](image)

**Answer**

Since pipes AB and BC are in series and water is incompressible, the volume flow rate will be the same in each pipe, $Q_1 = Q_2$. But

$$Q_2 = A_2 \cdot V_2 = (\pi/4) d_2^2 V_2$$

$$Q_1 = Q_2 = (\pi/4) (0.075)^2 V_2 = 8.836 \times 10^{-3} \text{ m}^3/\text{s}$$

$$V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{(\pi/4) d_1^2} = \frac{8.836 \times 10^{-3}}{(\pi/4) (0.05)^2} = 4.5 \text{ m/s}$$

Considering pipes BC, CD and DE, the discharge from BC must be equal to the sum of the discharges through CE and CE. Therefore

$$Q_2 = Q_3 + Q_4$$

And since

$$Q_4 = \frac{1}{2} Q_3$$
⇒ \( Q_2 = 1.5 \ Q_3 \)

Hence \( Q_3 = \frac{Q_2}{1.5} \)
\[
= \frac{8.836 \times 10^{-3}}{1.5} \\
= 5.891 \times 10^{-3} \text{ m}^3/\text{s}
\]

& \( Q_4 = \frac{1}{2} \ Q_3 \)
\[
= 2.945 \times 10^{-3} \text{ m}^3/\text{s}
\]

Also, since \( Q_3 = \frac{\pi}{4} \ d_3^2 \ v_3 \)

Hence \( d_3 = \sqrt{\frac{4Q_3}{\pi V_3}} \)
\[
= \sqrt{\frac{4 \times 5.891 \times 10^{-3}}{\pi \times 1.5}} \\
= 0.071 \text{ m}
\]

\( V_4 = \frac{Q_4}{(\pi/4) d_4^2} \)
\[
= \frac{2.945 \times 10^{-3}}{(\pi/4) \times 0.03^2} \\
= 4.17 \text{ m/s}
\]
5.3 Bernoulli’s Equation

Figure below shows a small streamtube element of cylindrical section with uniform cross-sectional area. The fluid accelerates in the direction of the flow. Taking this to be a control volume at an instant in the time,

Forces acting on the element in the s direction include:

(i) Pressure forces expressed as:
   upstream end \( p \cdot \Delta A \)
   downstream end \(- (p + \frac{\partial p}{\partial s} \Delta s) \Delta A \)
   (pressure may vary with respect to space, s and time, t, therefore the partial derivative is used).

(ii) Gravity force \( \Delta W \cdot \sin \alpha \) where \( \Delta W = \gamma \cdot \Delta s \cdot \Delta A \)

(iii) \( \sin \alpha = \frac{\Delta z}{\Delta s} \), then as \( \Delta s \) approaches zero, then \( \sin \alpha = \frac{\partial z}{\partial s} \)

By applying Newton’s second law in the s direction, the following equation is reached.

\[ \Sigma F_s = M \cdot a_s \]

where \( M = \) mass of the cylindrical element \( (p \cdot \Delta s \cdot \Delta A) \)
\( a_s = \) acceleration in the s direction
Therefore, we arrive at the following equation by substituting the above expression into the above equation.

\[- \frac{\partial}{\partial s} (p + \gamma z) = \rho \cdot a_s \quad - \text{Euler’s equation}\]

By rewriting the acceleration term in a general form as follows:

\[a_s = \frac{dV_s}{dt} = \frac{\partial V_s}{\partial t} + \frac{\partial V_s}{\partial s} \frac{ds}{dt}\]

For \textit{steady flow}, \[\frac{\partial V_s}{\partial t} = 0\]

Then \[a_s = V_s \frac{\partial V_s}{\partial s}\]

By substituting the above expression into Euler’s equation we obtain the following

\[- \frac{\partial}{\partial s} (p + \gamma z) = \rho \cdot V_s \frac{\partial V_s}{\partial s}\]

Assuming the \textit{density of the fluid remains unchanged},

\[\frac{\partial p}{\partial s} + \gamma \frac{\partial z}{\partial s} + \rho \frac{\partial}{\partial s} (V_s^2) = 0\]

By integrating the above equation with respect to \(s\) we have,

\[p + \gamma z + \frac{\rho}{2} V_s^2 = \text{constant}\]

or \[\frac{p}{\gamma} + z + \frac{V^2}{2g} = \text{constant}\]

The subscript \(s\) is usually omitted and \(v\) is used to represent the velocity in the direction of fluid flow.
The term \( \frac{p}{\gamma} \) - pressure head  
\( z \) - potential head (elevation head)  
\( \frac{V^2}{2g} \) - velocity head

The constant at the right hand side of the equation is the total head (total energy per unit weight) of the flow field. The equation is applicable when the flow is steady, non-viscous, and incompressible with the constant density. Under these conditions the total head at any point along a streamline of the flow field is the same.

Bernoulli’s equation is an expression of the principle of conservation of energy.

In most applications, Bernoulli’s equation is applied at two points in the flow field with \( z_1 \) and \( z_2 \) are referred to the same datum.

\[
\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}
\]
Worked examples:

1. Determine the velocity of the water flowing at point 1 and 2 of the pipe shown below, with the following data. Neglect all losses in heads.

\[ p_1 = 120 \text{ kPa}, \quad p_2 = 200 \text{ kPa}, \quad d_1 = 0.4 \text{ m} \quad \text{and} \quad d_2 = 0.6 \text{ m} \]

Answer

Apply Continuity equation to point 1 and 2

\[ V_1 A_1 = V_2 A_2 \]

Since \[ A_1 = \frac{\pi (0.4)^2}{4} \text{ m}^2, \quad A_2 = \frac{\pi (0.6)^2}{4} \text{ m}^2 \]

\[ \therefore V_1 = V_2 \cdot \left(\frac{0.6}{0.4}\right)^2 \]

\[ = 2.25 V_2 \]

Apply Bernoulli’s equation along the central streamline to points 1 and 2

\[ \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \]

\[ p_1 = 120 \text{ kPa}, \quad p_2 = 200 \text{ kPa} \]

\[ V_1 = 2.25 V_2 \]

& \[ z_1 = z_2 \text{ (same level)} \]

Hence

\[ \frac{120}{9.81} + \frac{(2.25V_2)^2}{2 \cdot 9.81} = \frac{200}{9.81} + \frac{V_2^2}{2 \cdot 9.81} \]

\[ \Rightarrow V_2 = 6.3 \text{ m/s} \]

and

\[ V_1 = 2.25 v_2 \]

\[ = 14.2 \text{ m/s} \]
2. For a 50 mm diameter siphon drawing oil (S.G. = 0.82) from the oil reservoir as shown below, the head loss from point 1 to point 2 is 1.5 m and from point 2 to point 3 is 2.40 m. Find the discharge of oil from the siphon and the oil pressure at point 2.

Answer

Apply Bernoulli’s equation to points 1 & 3.

\[
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 + \text{loss}_{1-3}
\]

\[p_1 = p_3 = 0 \quad (p_{\text{atm}} = 0)\]
\[V_1 = 0\]

(Assume the oil vessel is very large, the rate of the oil level drop is negligible)

\[z_3 = 0, \quad z_1 = 5 \text{ m}\]
\[\text{loss}_{1-3} = \text{loss}_{1-2} + \text{loss}_{2-3} = 1.5 + 2.4 \text{ m} = 3.9 \text{ m}\]

\[
\therefore 0 + 0 + 5 = 0 + \frac{V_3^2}{2*9.81} + 0 + 3.9
\]
\[V_3 = 4.646 \text{ m/s}\]

\[Q = A_3 * V_3 = [\pi(0.05)^2/4]*4.646 = 0.00912 \text{ m}^3/\text{s}\]
Again, apply Bernoulli’s equation to points 1 & 2

\[
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \text{loss}_{1,2}
\]

\[
z_2 = 7 \text{ m}
\]
\[
V_2 = V_3 \quad \text{(same pipe diameter)}
\]
\[
= 4.646 \text{ m}
\]
\[
\text{loss}_{1,2} = 1.5 \text{ m}
\]

\[
0 + 0 + 5 = \frac{p_2}{\gamma} + \frac{(4.646)^2}{2\times9.81} + 7 + 1.5
\]
\[
\frac{p_2}{\gamma} = -4.6 \text{ m}
\]
\[
p_2 = -4.6 \times 9.81 \times 0.82 \quad \text{kPa}
\]
\[
= -36.9 \text{ kPa}
\]
5.4 Flow Measurement

The three parameters most often involved in flow measurement are pressure, velocity and discharge of flow. The relationships between these parameters have already been defined through the Continuity and Bernoulli’s equation. In this section, the principles and applications of common flow measurement devices are discussed.

5.4.1 Stagnation Tube and Pitot Tube

When an L shaped tube is placed in a flow field, it can be used to measure the velocity of flow.

\[ V = \sqrt{2gh} \]

- Torricelli’s formula

where \( V \) = velocity of flow
\( h \) = the level of fluid in the tube above the free surface.

This expression can be derived by applying Bernoulli’s equation to the stagnation point and any other point upstream of it. For simplicity, choose a point (point A) at the same level as the stagnation point (point B, i.e. the velocity of fluid flow at this point is zero).
From Bernoulli’s equation,

\[ \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \]

Since \( z_A = z_B \), and \( V_B = 0 \) (stagnation point), therefore

\[ \frac{V_A^2}{2g} = \frac{p_B}{\gamma} - \frac{p_A}{\gamma} \]

which can be applied to flow in closed or open conduits.

Since also \( p_A = \gamma z \) and \( p_B = \gamma (z + h) \), then

\[ V_A = \sqrt{2gh}, \]

where \( V_A \) = the velocity of flow which is measured by the stagnation tube

Since the application of the stagnation tube is limited by the pressure of the flow, pitot tubes are often used in pressure pipes.
**Worked examples:**

1. Water flows through the pipe contraction shown in figure below. For the given 0.2 m difference in manometer level, determine the flow rate when the diameter of the small pipe, D is 0.05m.

![Pipe Contraction Diagram](image)

**Answer**

Again, apply Bernoulli’s equation to points 1 & 2

\[
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2
\]

\[V_1 = 0 \quad \text{(stagnation point)}\]
\[z_1 = z_2 = 0 \quad \text{(same level)}\]

\[\therefore \quad \frac{p_1}{\gamma} + 0 + 0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + 0\]

\[V_2 = \sqrt{\frac{2g(p_1 - p_2)}{\gamma}}\]

but \(p_1 = \gamma h_1\) and \(p_2 = \gamma h_2\)

\[\therefore \quad p_1 - p_2 = \gamma(h_1 - h_2)\]

\[= 0.2 \gamma\]

Thus \[V_2 = \sqrt{\frac{2g \cdot 0.2\gamma}{\gamma}} = \sqrt{2g \cdot 0.2} \text{ m/s} = 1.98 \text{ m/s}\]

\[Q = A_2 \cdot V_2\]

\[= \pi(0.05)^2/4 \cdot 1.98\]

\[= 3.888 \cdot 10^{-3} \text{ m}^3/\text{s}\]

\[= 3.888 \text{ L/s}\]
2. Water is siphoned from a tank as shown below. Determine the flow rate and the pressure at point A, a stagnation point.

![Diagram of water siphoning](image)

**Answer**

Apply Bernoulli’s equation to points 1 & 2

\[
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2
\]

\[p_1 = p_2 = 0 \quad (p_{\text{atm}})\]
\[V_1 = 0\]
\[z_1 = 3\text{m}, \quad z_2 = 0\]

\[\therefore \quad 0 + 0 + 3 = 0 + V_2^2/(2*9.81) + 0\]

\[V_2 = \sqrt{2*9.81*3} \quad \text{m/s}\]

\[= 7.67 \text{ m/s}\]

Hence \[Q = A_2 * V_2\]

\[= [\pi(0.04)^2/4]*7.67 \quad \text{m}^3/\text{s}\]

\[= 9.64 * 10^{-3} \text{ m}^3/\text{s}\]

Again, apply Bernoulli’s equation to points 1 & A

\[
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A
\]

\[V_A = 0 \quad \& \quad z_A = 0\]

\[0 + 0 + 3 = p_A/\gamma + 0 + 0\]

or \[p_A = \gamma* 3\]

\[= 9.81 * 3 \quad \text{kPa}\]

\[= 29.43 \text{ kPa}\]
5.4.2 **Orifice**

An orifice is a geometric opening in the side of a wall and tank, through which fluid can flow. A circular sharp edged orifice at the side of a water tank is shown.

The volume rate of flow discharged through an orifice will depend upon the head of the fluid above the level of the orifice and it can therefore be used as a means of flow measurement.

Applying Bernoulli’s equation to A and B, assuming that there is no loss of energy,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

Putting $z_A - z_B = H$, $V_A = 0$, $V_B = V$ and $p_A = p_B$,

velocity of jet, $V = \sqrt{2gH}$

Theoretically, if $A$ is the cross-sectional area of the orifice,

Discharge, $Q = \text{Area} \times \text{Velocity}$

$$= A \sqrt{2gH}$$

In practice, the actual discharge is considerably less than the theoretical discharge and is modified by introducing a *coefficient of discharge*, $C_d$, so that

Actual discharge, $Q_{\text{actual}} = C_d \times Q_{\text{theoretical}}$

$$= C_d A \sqrt{2gH}$$
There are two reasons for this phenomenon. First, the velocity of the jet is less than that of the theoretical velocity because there is a loss of energy between A and B:

$$\text{Actual velocity at B} = C_v \times V = C_v \sqrt{2gH}$$

where $C_v = \text{coefficient of velocity} \approx 0.97$

Second, the paths of the particles of the fluid converge on the orifice and the area of the issuing jet at B is less than the area of the orifice A at C. In the plane of the orifice, the particles have a component of velocity towards the centre and the pressure at C is greater than atmospheric pressure. The streamlines will converge at a short distance downstream of the orifice and the minimum flow area is called the vena contracta. This is smaller than the cross-sectional area of the orifice.

The ratio of the cross-sectional area of the vena contracta to that of the orifice is called the coefficient of contraction, $C_c$.

$$\text{Actual area of the jet at B} = C_c A$$

$$\therefore \text{Actual Discharge} = \text{Actual area at B} \times \text{Actual velocity at B} = C_c A \times C_v \sqrt{2gH} = C_c \times C_v A \sqrt{2gH}$$

Hence $C_d = C_c \times C_v$
Evaluation of the actual velocity from the displacement of the jet

For horizontal motion,

\[ S = x, \quad u = V, \quad \text{and} \quad a = 0 \]

Hence

\[ x = V \times t \]

where \( V = \) actual velocity at vena contracta

For vertical motion,

\[ u = 0, \quad a = g \]

\[ y = \frac{1}{2} g \times t^2 \]

By eliminating \( t \),

\[ V = \sqrt{\frac{g x^2}{2y}} \]
**Worked examples:**

1. Oil of specific gravity 0.82 discharges from an open tank through an orifice of diameter 14 mm. The coefficient of velocity is 0.88 and the coefficient of contraction is 0.62. The centre of the orifice is at a depth of 0.9m from the surface of the oil. Determine the diameter of the *vena contracta* and the discharge of oil through the orifice.

Answer

\[
A = \pi \times (14/1000)^2/4 = 1.54 \times 10^{-4} \text{ m}^2 \\
\text{Since } C_c = \frac{A_c}{A} \\
0.62 = \frac{A_c}{1.54 \times 10^{-4}} \\
\text{hence } A_c = 0.955 \times 10^{-4} \text{ m}^2 \\
\text{or } d_c = 0.01102 \text{ m } = 11.02 \text{ mm} \\
\]

Theoretical velocity, \( V = \sqrt{2gh} \) 
\[
= \sqrt{2 \times 9.81 \times 0.9} = 4.0202 \text{ m/s} \\
\]

Theoretical discharge, \( Q = V \times A \) 
\[
= 4.02 \times 1.54 \times 10^{-4} = 6.47 \times 10^{-4} \text{ m}^3/\text{s} \\
\]

Coefficient of discharge, \( C_d = C_v \times C_c \) 
\[
= 0.88 \times 0.62 = 0.546 \\
\]

Actual discharge = \( C_d \times \) Theoretical discharge 
\[
= 0.546 \times 6.47 \times 10^{-4} = 3.53 \times 10^{-4} \text{ m}^3/\text{s} \\
= 0.353 \text{ L/s} \\
\]
2. If viscous effects are neglected and the tank is large, determine the flow rate from the tank as shown below. (Neglect all losses)

![Diagram of fluid system with orifice and liquid levels]

**Answer**

Apply Bernoulli’s equation between to 2 & 3

\[
\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3
\]

- \(p_2 = p_1 + \gamma_{oil}h = 2\gamma_{oil}\) (\(p_1 = p_{atm} = 0\))
- \(p_3 = 0\) (\(p_{atm}\))
- \(V_2 = 0\)
- \(z_2 = 0.7\) m \& \(z_3 = 0\)

Thus

\[
\gamma_{oil}/\gamma_{w} + 0 + 0.7 = 0 + V_3^2/(2g) + 0
\]

\[
V_3 = \sqrt{2g \times \frac{2\gamma_{oil}}{\gamma_{w}} + 0.7} \text{ m/s}
\]

Since \(S.G_{oil} = \gamma_{oil}/\gamma_{w} = 0.81\)

\[
V_3 = \sqrt{2g \times (2 \times 0.81 + 0.7)} \text{ m/s}
\]

\[
= 6.747 \text{ m/s}
\]

\[
Q = A_3 \times V_3
\]

\[
= \pi(0.05)^2/4 \times 6.747 \text{ m}^3/\text{s}
\]

\[
= 0.0132 \text{ m}^3/\text{s}
\]
5.4.3 Venturi Meter

The venturi meter is the most commonly used device for flow measurement and is designed with a streamlined throat which can,

1. reduce the overall head loss, and
2. reduce the normal mechanical wear of the device. The discharge equation for a venturi meter is the same as for an orifice meter, except that the flow coefficient value of a venturi meter is generally higher than that of an orifice meter and a flow nozzle. This follows since the coefficient of contraction of a venturi meter is unity.

![Venturi Meter Diagram](image)

Applying Bernoulli’s equation with $p_1 = \rho gh_1$, $p_2 = \rho gh_2$ and $z_1 = z_2$, and also by continuity equation, $V_1 A_1 = V_2 A_2$

$$V_1 = \sqrt{\frac{2gh_1 - h_2}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

Hence discharge $Q = A_1 V_1$

$$Q = A_1 \frac{A_1}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}} \sqrt{2gh_1 - h_2}$$

Actual discharge $= C_d Q$

$$= C_d A_1 \frac{A_1}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}} \sqrt{2gh_1 - h_2}$$
Worked examples:

1. The flow of kerosene is measured using a venturi meter. The diameter of the pipe and the throat are 50 mm and 25 mm respectively. A differential manometer shows a deflection, $h'$ of 55 mm of mercury. The coefficient of discharge is 0.96. Determine the flow rate of kerosene. Take the S.G. of kerosene as 0.82.

Answer

Area of cross-section of the pipe, $A_1 = \pi \times \frac{0.05^2}{4} = 0.00196$ m$^2$
Area of cross-section of the throat, $A_2 = \pi \times \frac{0.025^2}{4} = 0.00049$ m$^2$

By continuity equation, $V_2 = \left( \frac{A_1}{A_2} \right) * V_1 = \left( \frac{0.00196}{0.00049} \right) * V_1 = 4V_1$

By Bernoulli’s equation,

\[
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2
\]

\[
0.857 + \frac{V_1^2}{2g} = \frac{(4V_1)^2}{2g}
\]

\[
0.857 + \frac{V_1^2}{2g} = \frac{16V_1^2}{2g}
\]

or $V_1 = 1.059$ m/s

$V_2 = 4 \times 1.059 = 4.235$ m/s

$Q'_{actual} = 0.96 \times 4.235 \times 0.00049 = 0.00199$ m$^3$/s = 1.99 L/s
2. The water supply to a gas water heater contracts from 10 mm in diameter at 1 to 7 mm in diameter at 2 (figure below). If the pipe is horizontal, calculate the difference in pressure between 1 and 2 when the velocity of the water at 1 is 4.5 m/s. The pressure difference operates the gas control through connections which are taken to horizontal cylinder in which a piston of 20 mm diameter moves. Ignoring friction and the area of the piston connecting rod, what is the force on the piston?

![Diagram of water supply and gas water heater](image)

Answer

For Continuity of flow,

\[ V_1 A_1 = V_2 A_2 \]

Since \( A_1 = \frac{\pi (0.1)^2}{4} \) m², \( A_2 = \frac{\pi (0.07)^2}{4} \) m²

\( V_1 = 4.5 \) m/s

\[ V_2 = 4.5 \times \frac{(0.1)^2}{(0.07)^2} \text{ m/s} \]

\[ = 9.18 \text{ m/s} \]

Apply Bernoulli’s equation to points 1 & 2

\[ \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \]

\[ z_1 = z_2 \quad \text{(same level)} \]

\( V_1 = 4.5 \text{ m/s} \)

\( V_2 = 9.18 \text{ m/s} \)

hence

\[ \frac{p_1}{\gamma} + \frac{4.5^2}{2 \times 9.81} + 0 = \frac{p_2}{\gamma} + \frac{9.18^2}{2 \times 9.81} + 0 \]
\[
\frac{p_1 - p_2}{\gamma} = \frac{9.18^2 - 4.5^2}{2 \times 9.81} \\
= 3.267 \text{ m of water}
\]

\[
p_1 - p_2 = 3.267 \times 9.81 \text{ kPa} \\
= 32.045 \text{ kPa}
\]

Area of piston, \(A_p = \frac{\pi(0.02)^2}{4} \text{ m}^2 \\
= 0.000314 \text{ m}^2
\]

Force on piston = \((p_1 - p_2)A_p \\
= 32.045 \times 0.000314 \text{ kN} \\
= 0.01 \text{ kN}

3. Air flows through the device shown below. If the flow rate is large enough, the pressure within the construction will be low enough to draw the water up into the tube. Determine the flow rate, Q and the pressure needed at section 1 to draw the water into section 2. Neglect compressibility and viscous effects.

![Diagram of the device](image)

Answer

For Continuity of flow,

\[ V_2 A_2 = V_3 A_3 \]

\[ \therefore V_2 = V_3 \frac{(d_3)^2}{(d_2)^2} \]

\[ = V_3 \frac{(50)^2}{(25)^2} \]

\[ = 4V_3 \]

Apply Bernoulli’s equation between to 2 & 3

\[ \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \]

\[ p_3 = 0 \]

\[ z_2 = z_3 \text{ (same level)} \]

\[ p_2 = -\gamma w h \]

\[ \frac{p_2}{\gamma} = -\frac{\gamma w h}{\gamma} \]

\[ = -\frac{9.81*1000}{12}*0.3 \text{ m} \]

\[ = -245.25 \text{ m} \]
Thus  
\[-245.25 + \frac{(4V_3)^2}{2g} + 0 = 0 + \frac{V_3^2}{2g} + 0\]

\[\frac{(4^2 - 1) V_3^2}{2} = 245.25 \times 2 \times 9.81\]

\[V_3 = 17.91 \text{ m/s}\]

\[Q = A_3*V_3\]

\[= \pi(0.05)^2/4*17.91 \quad \text{m}^3/\text{s}\]

\[= 0.0351 \text{ m}^3/\text{s}\]

Also apply Bernoulli’s equation between to 1 & 3

\[\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3\]

\[V_1 = V_3 \quad \text{(same diameter)}\]

\[z_1 = z_3 \quad \text{(same level)}\]

therefore  

\[p_1 = p_3 = 0 \quad (P_{\text{atm}})\]
Class Exercise 5.1:

Water flows down the ramp in the channel as shown below. The channel width decreases from 4.5 m at section 1 to 2.5 m at section 2. For the conditions shown, determine the flow rate.

\[(14.12 \text{ m}^3/\text{s})\]
Class Exercise 5.2:

The water in a tank is 1.8 m deep and over the surface is air at pressure 70 kPa (gauge). Find the flow rate from an orifice of 50 mm in the bottom of the tank if the $C_d = 0.6$

$$\text{orifice dia} = 50 \text{ mm}$$

$$1.8 \text{ m}$$

$$70 \text{ kPa}$$

$$0.0156 \text{ m}^3/\text{s}$$
Class Exercise 5.3:

A horizontal venturi tube, 280 mm diameter at the entrance and 140 mm diameter at the throat, has a discharge coefficient of 0.97. A U-tube manometer filled with mercury is connected between the entrance and the throat to measure the water flow between them. Calculate the flowrate when the difference in the mercury level is 50 mm. (0.0542 m³/s)
**Tutorial: Energy Equation**

1. Water flows from the faucet on the first floor of the building shown below with a maximum velocity of 6.5 m/s. For steady inviscid flow, determine the maximum water velocity from the basement faucet and from the faucet on the second floor. Assume only two taps will be opened at each time and each floor is 4 m tall.

   ![Diagram](image1)

2. For the water shooting out of the pipe and nozzle under the conditions shown in figure below, find the height, h about the nozzle to which the water jet will shoot. Assume negligible head loss.

   ![Diagram](image2)

3. Oil (S.G = 0.86) flows through the system as shown in figure below. A manometer is used to measure the flowrate of the oil. By neglecting any losses, determine the flowrate of the oil.

   ![Diagram](image3)