Double Integration

Iterated integrals
1. Calculate the value of the following iterated integrals:
   (a) \[ \int_{0}^{1} \int_{-1}^{2} (x^2 + y^2) \, dx \, dy \]
   (b) \[ \int_{-1}^{2} \int_{0}^{1} (x^2 + y^2) \, dy \, dx \]

   What conclusion can you draw from the results of part (a) and (b)?

Double integrals
2. By integrating with respect to \( x \) first and \( y \) second, evaluate the double integral \( \iint_{R} xy \, dA \), where \( R \) is the region bounded by the curves \( y = x \) and \( y = x^3 \).

3. "The double integral can be evaluated by integrating either with respect to \( x \) first and \( y \) second or vice versa." Do you agree with this statement?

   Justify your answer by evaluating the integral \( \iint_{R} e^{-x} \, dA \), where \( R \) is the region in the \( x \)-\( y \) plane bounded by the lines \( y = x, y = 0, \) and \( x = 1 \).

Finding volumes
4. (a) Show how to use an iterated integral of the form \( \int_{a}^{b} \int_{0}^{h(y)} f(x, y) \, dx \, dy \) to find the volume of the solid that lies under the surface \( y = e^{-x} e^{-y} \) and over the triangular region \( R \) with vertices \((0,0), (1,0), (0,1)\).

   (b) Find the volume of the solid in part (a) by evaluating an iterated integral of the form \( \int_{a}^{b} \int_{0}^{h^{-1}(x)} f(x, y) \, dy \, dx \).

Finding areas
5. (a) Find the points of intersection of the line \( y = 2x \) and the parabola \( y^2 = 16x \).

   (b) Find, by a double integral, the area enclosed by \( y = 2x, y^2 = 16x \) and the ordinate at \( x = 1 \).

Finding mass, centroid and moments of inertia
6. Evaluate the mass, centroid and moment of inertia about \( x \) and \( y \) of an object is the density function, \( \rho(x,y) = x \) and the area is enclosed by \( x = 0, y = a, \) and \( y = x \).